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## Learning Mathematics Through the Modern Didactic Principle of Polyformity

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### Abstract

Modern teaching of mathematics tries to eliminate the mechanical memorization of a large amount of knowledge, and also tends to avoid the formal development of students' psychological abilities. It is normal that such teaching implies the development of student abilities, not on worthless mathematical material, but on content that is of high quality in terms of education and training. Solving problems, which arouses students' interest and triggers ingenuity, producing experiences of the tension of self-engagement, as a result, necessarily has the triumph of the inventor. Such experiences can create a tendency for mental work, making an indelible mark on the spirit and character of a young person.

**Keywords:** goal of learning mathematics, didactic principle of polyformity, teacher's role

### Introduction

The very concept of mathematics is in frequent use in various social spheres. As a teaching subject, it activates logical thinking in students and thereby contributes to the development of their competences. In this way it prepares them for certain life vocations and choosing their future occupations. Mathematics teaches us to be wise and to solve problems step by step. It should be loved, because it is logically tangled and as such presents a challenge to many people and encourages their readiness to deal with it in different ways. Each of us can learn math. There is no human activity that does not depend on mathematics. All secondary schools, and many faculties have mathematics as a subject. The importance of mathematics is enormous, not only as an educational subject, but also something that contributes to the development of per-

sonality. The goals and tasks of the teaching subject are strictly defined, and in elementary school they are intended for the acquiring of basic linguistic and mathematical literacy. During schooling, 15% of students express a greater or lesser need for private mathematics lessons by the time they enter the fifth grade of elementary school, 25% by the time they enter high school, 40% by the time they enter college, and up to 90% at the faculties where they study, or more precisely, listen to this subject. Many researches conducted in Serbia in the last few years point to a high level of mathematical anxiety among students. More than half of students worry that they will have difficulties in maths classes and that they will get bad grades. Indifference, if not resistance to mathematics is, in essence, a fundamental trait of human beings. That limitation should be taken into account and adapted to, instead of teachers deluding themselves that a suitable presentation will make mathematics interesting. Mathematics is a science that was created by studying figures and calculating with numbers. It is a science that studies structures that it creates itself or which originate from other sciences (most often physics, but also other natural and social sciences) and describes the properties of those structures. However, mathematics can tell us many things. There are many prejudices about mathematics. Unlike other subjects, in school mathematics continuity is necessary, it is necessary to work constantly. Today's students, unfortunately, are generally not inclined to work constantly, they are not persistent enough. The role of teachers is also important here, their readiness to help and their knowledge, ability and talent to do it in the right way. A large part of the material will never be used by most students, although some students will use it all the time. No one can predict exactly how useful knowledge of the square root algorithm will be in the future. It will probably be unnecessary, but what if it inspires someone for some future projects? The number of good ideas is surprisingly small and most of them, after failing in one form, reappear in new forms. "Unnecessary" ideas that are on the decline today may be a hit again when today's students are at the peak of their careers. And all of them can feel the consequences of the hasty reduction of materials due to someone's illusion that the future can be reliably predicted.

### **The subject matter of the study. The Goal of Learning Mathematics**

What distinguishes mathematical thinking from social and natural sciences is not its flexibility, activity, orientation, economy, depth, breadth, originality, laconicism, etc., but its characteristic operation with abstractions, i.e. mathematical phenomena, objects, which are devoid of any materiality, and which we can present in various ways, while only the relations between them are immutable, i.e. invariant. Of course, those relations are described by mathematical axioms. However, if we keep part of the mathematical phenomena (objects) and add to them either real or some other abstract objects and apply to them that

characteristic operation with abstractions, whereby the relations between them remain invariant, and which we define partly by mathematical and partly by painting rules, then we get a complete a new mathematical expression, i.e. polyform mathematical style (Nikolic, Lipovac, Medic, 2022). As we have already mentioned, knowledge of mathematics does not improve thinking, although people who think precisely are usually good at mathematics. But that connection is a result of an innate tendency for math/precision thinking, not an effect of math knowledge on thinking. This idea, about the global effects of mathematical knowledge on other areas of knowledge and life, is part of the discredited doctrine of “knowledge transfer”, which assumes that acquired knowledge is easily generalized and transferred from domain to domain, so for example when a person has mastered the material of formal logic well, that person becomes a master in logical reasoning on every topic. People think concretely, not abstractly, and it is very difficult for them to apply acquired knowledge to new contexts. Depending on the school’s goal for students to learn the application of mathematics, in most cases this application must be specially practiced, because it cannot be assumed that it arises spontaneously from the knowledge of mathematics. This also means that students should be specially prepared for all international mathematics tests based on the application of mathematics and not for dealing with mathematical concepts. However, the application of mathematics must not replace knowledge of mathematics, in the sense of dealing with mathematical concepts and techniques, because the purpose of knowledge of mathematics is only that knowledge, which is a necessary condition for all engineering, a large number of natural and increasingly social sciences. For example, one who does not know algebra cannot master differential calculus, and one who cannot master it, can, for the beginning, say goodbye with practically all engineering professions, a great number of natural sciences and economics. It also permanently limits his understanding of any field that uses statistics. Mathematical literacy is one of the key competencies for lifelong learning, defined as the ability to develop and apply mathematical thinking in order to solve a series of problems in everyday situations (Anić, Pavlović-Babić, 2011) and to ensure that all students acquire basic language and mathematical literacy and progress towards the realization of the appropriate Educational Achievement Standards, as well as to:

- enable students to solve problems and tasks in new and unfamiliar situations,
- enable students to express and justify their opinion and discuss with others,
- develops motivation for learning and interest in subject contents,
- ensure that students acquire elementary mathematical knowledge that is necessary for understanding phenomena and laws in nature and society,
- train students to apply acquired mathematical knowledge in solving various tasks from life practice,

– represents the basis for successful continuation of mathematics education and for self-education,

– contributes to the development of mental abilities, the formation of a scientific view of the world and the all-round development of the student's personality.

In the Republic of Serbia, basic education and upbringing is acquired at school through the implementation of a curriculum lasting eight years. Classroom teaching is organized for students from I to IV grades, and subject teaching is organized for students from V to VIII grades. Teaching subjects in the curriculum are distributed during schooling by classes, and they can be compulsory or optional. Mathematics as a teaching subject appears in ancient Greek schools under the name *logistica numerosa*. Children in age of today's primary school age learned, with the help of an abacus, to count with numbers, which was useful in their everyday life. Geometry was then a scientific discipline that was studied in classical schools of philosophy and mathematics. Therefore, in the teaching of mathematics, the principle of polyformity should have a universal role, which would be presented by enriching the teaching with various contents, means, procedures and methods (Marković, 2012). Today, mathematics is a compulsory general educational subject, whose position in relation to other subjects is indicated by the fact that from I to IV grades, mathematics is offered five hours a week, while in higher grades the number of hours is reduced to four hours a week. If we look at the upper grades of elementary school, except for Serbian language classes in the fifth grade, which is represented by five classes a week, mathematics together with the Serbian language has the maximum number of classes. Mathematics in elementary school is determined by contents, goals and tasks that are determined by the age and psychophysical capabilities of the students. The transfer of knowledge in class was created with the aim of leading the student to the correct conclusion and knowledge through the application of appropriate teaching forms, methods and tools. As mathematics is increasingly applied in everyday life, its successful application requires a general mathematical education. The new principle of polyformity of school organization is based on global requirements for effective pedagogical standards, which enable lifelong learning. Traditional value systems, which, until recently, reflected different patterns of living environments, are increasingly being imposed by a globally acceptable standard of education in modern schools (Nikolić, Hilčenko, 2021).

### **Research methodologies and tools. Didactic Principle of Polyformity**

The didactic principle of polyformity is not encountered at all as a didactic peculiarity, and if it is applied in some places, then it is very rare, intuitive, spontaneous, singular and accidental in the teaching of mathematics in primary

school, high school and colleges. The essence of the application of this principle consists in the permanent insistence on the integral consideration of various approaches to the comprehension and understanding of the studied teaching phenomena, therefore its exploitation in practice requires from the teacher excellent knowledge and the skill of applying the most diverse professional-didactic-methodical possibilities, and induces intensive thinking activity of students expressed in high-quality self-exalting work and greater motivation. (Marković, 2008). The effectiveness of the principle of polyformity is based on the evident psychological fact that changes and diversity in work refreshes teaching, and monotony generally induces a weakening of interest and the appearance of passivity and boredom. The principle of polyformity, due to the mentioned peculiarities, represents a universal scientific and teaching principle, whose epistemological basis is identical to that of the principle of permanence, the law of negation of negation, whereby the principle of polyformity takes on the characteristics of a dialectical law. As the principle of polyformity includes all existing didactic principles, it elevates this principle to the pedestal of universality. That is why the principle of polyformity should play a universal role in teaching mathematics, which would be presented by enriching the teaching with various contents, means, procedures and methods. When we talk about content, we mean the selection of such tasks that allow for a greater number of diverse approaches to solving them and using obvious means. However, organizing such classes requires an adequate application of the polyformity of methodical forms and methodical details of teaching, i.e. their variations in the same lesson (Nikolić, 2021).

### **Analysis of research results. Application of the Principle of Polyformity in Mathematics Teaching**

The essence of the didactic principle of polyformity is reflected in the permanent insistence on the integral consideration of diverse approaches to comprehension and understanding of the studied teaching phenomena. Its exploitation in practice requires the teacher to have excellent knowledge and the ability to perceive the most diverse professional-didactic-methodical possibilities, and it induces an intensive thinking activity of the students made by high-quality self-examination work and greater motivation. That is why the principle of polyformity should have a universal role in teaching mathematics, which would be presented by enriching the teaching with various contents, means, procedures and methods (Penavin, 1971). When we talk about content, we mean the selection of such tasks that allow for a greater number of diverse approaches to their solution and the use of obvious means. However, the organization of such classes requires an adequate application of the polyformity of methodical forms and methodical details of teaching, i.e. Their variations, even methodological innovations in the same lesson. The methodical forms and methodical details that

the teacher plans and observes during the lesson are based on the timely pulsation of didactic principles, which is manifested in their simultaneous polyform-cohesion effect, i.e. integral dialectical unity.

### **The Role of the Teacher**

Learning math is not fun for many people. Mathematics for children of early age was “fun because they added numbers”. It was also interesting “when we drew some boots and then added them together”, as the children say. The reason why math is boring is, basically, that humans didn’t evolve in modern cities but rather in small, technologically primitive groups. As a result, natural selection favored those traits that provided an evolutionary advantage in that environment – traits that may or may not be adaptive today. For example: for someone living in a small group of people, knowing information about who did what to whom is of fundamental importance, and this is the root of the universal human need for gossip (in the modern environment, this instinct has the unusual consequence that people collect information about celebrities – people with whom they have no connection, but who are perceptibly constantly present, which once upon a time meant really relevant). Thus, indifference if not resistance to mathematics is, in essence, a fundamental trait of human beings. And that limitation should be taken into account and adapted to, instead of deluding yourself that mathematics can become interesting with a suitable presentation. Like writing, mathematics is a laboriously devised cultural product rather than a matter of natural instinct. Except in the case of rare individuals, it will not inspire enthusiasm, although long, regular and thorough work can make its learning less painful. All this doesn’t mean that the way mathematics is taught is completely irrelevant. There are good and bad math teachers, but the quality of a teacher is not how well he is able to entertain the student, but how well he is able to teach him math.

The basic conditions for this are that:

- the teacher himself is good at mathematics,
- the teacher is able to understand what the students do not understand.

The problem in mathematics arises primarily due to omissions in the previous school material. In an ideal case, the teacher is able to identify the specific failure, separate the student’s knowledge from ignorance, and using the student’s islands of knowledge (which, as a rule, exist even in the worst students) repair the damaged foundations and gradually incorporate new school material.

### **Problems in Training Procedures**

Problems in the training process have a special importance for the teacher himself. They are mainly related to the simultaneous speed and the large number of demands placed on the student. According to Vladislavljević (1986), this procedure contains processes of analysis and synthesis. In the physiological


sense, both processes are very complicated and the transformation of one process into another, then into a third, is carried out gradually, all the more slowly if the child is younger. For example, when reading, there must be compatibility of visual, acoustic and articulatory processes. In this complex integrated circuit with a large number of functions, errors may occur in certain parts of the system when transmitting a message. It can be wrongly seen, or if the visual representation is well transferred to the auditory one, the auditory process can make a mistake in perception, and therefore gives a wrong command to the speech organs (or hand), etc. This is why mistakes occur. In order to reduce the possible or resulting difficulties, it is necessary to simplify and slow down the training process as much as possible, remove the multitude of impressions and the multitude of requirements, in order to create a single, but clean and safe engram (a hypothetical permanent change that occurs on the tissue of living organisms as a result of the action of external impact). Teaching mathematics has always been problematic. This applies to students in the classroom as well. Problems arise at the moment when abstraction exceeds the ability of students to understand the problem, that is, to move from manipulative/concrete thinking to abstract thinking. This happens because the transition in content and work methodology is too fast and unadjusted with the pace of development in students' learning from visual/motor to abstract thinking. These problems manifest in content such as units of measure, inequalities, fractions or word problems. Problems of this type can be greatly overcome if we present dry word problems in a more interesting way. When learning by way of self-aware polyform heuristics, as a dominant method within the polyform principles of interactive teaching, the content that students should learn is not presented in a finished form, but must be discovered, preferably in different ways. Then the students' intellectual power, motivation, activity in learning increases, and, due to the completed work, the feeling of satisfaction appears. Learning through the method of self-aware polyform heuristics has greater effects in terms of the acquiring of content knowledge, and especially process, i.e. applicable knowledge in the sense of modern taxonomies of knowledge, because the student makes his own efforts to organize newly acquired information, in his own information system, and to find the entire range of information he needs, which increases his ability to organize and arrange data, using deductive, analytical-synthetic approaches and the application of the same in various problematic and life situations (Nikolić, 2016). According to numerous researchers, modern teaching, which is a combination of principled and methodical "knitting", with the help of computers, and which is not known or recognized by traditional teaching, contains new qualities of diverse teaching work, increases the activity of students in the process of teaching and acquiring knowledge, affects their greater motivation, initiative, creativity and applicability of acquired knowledge in everyday life, which are the main goals of modern mathematics teaching.

The essence of this significant didactic principle is reflected in the permanent insistence on the integral consideration of various approaches to comprehending and understanding the studied teaching phenomena.

### Examples of interesting problems in mathematics

1) Task:

**Which **CODE** will open the lock?**



6	8	2
6	1	4
2	0	6
7	3	8
8	7	0

One number is correct and in the right place.

One number is correct, but it is not in the right place.

Two numbers are correct, but not in the right places.

There are no exact numbers.

One number is correct, but it is not in the right place.

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7	3	8
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There are no exact numbers.

8	7	0
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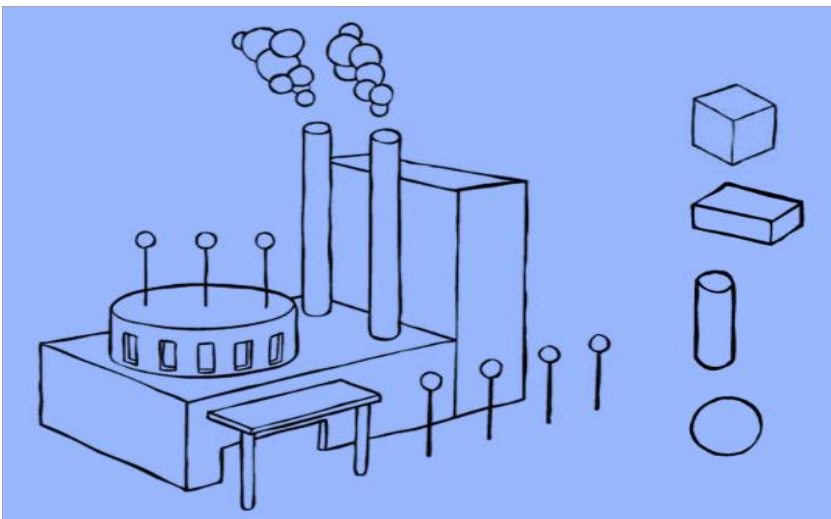
One number is correct, but it is not in the right place.

**CODE**


Okmatematics

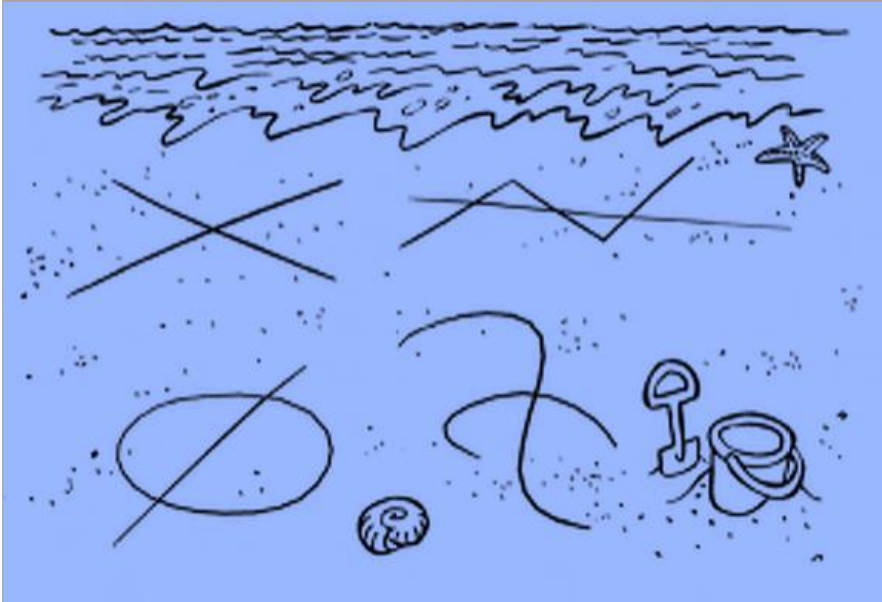
Solution: 042

2) Task: circle the geometric bodies seen in the picture





3) Task – mark the places where the lines intersect

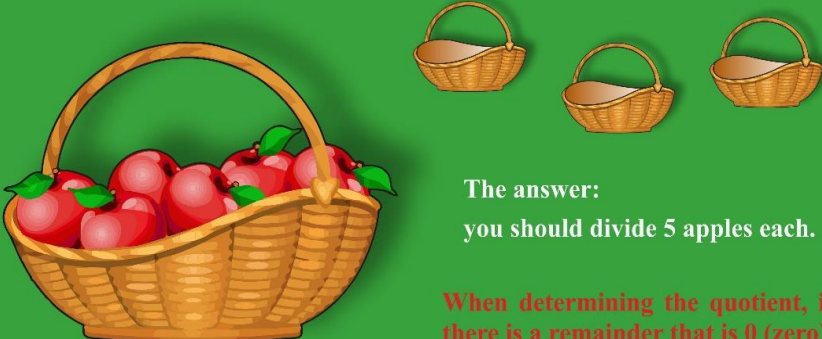


4) Task – let the skier connect these three flags with straight lines



5) Task – There are 15 apples in a large basket

There are 15 apples in a large basket.  
How many apples should be divided into three smaller baskets.

$$15 : 3 = 5$$


The answer:  
you should divide 5 apples each.

*When determining the quotient, if there is a remainder that is 0 (zero), then we say that the numerator is divisible by the divisor.*

### Conclusion

The acquisition of mathematical contents by some students is accompanied by discomfort and fear. For many of them, mathematics is one of their least favorite subjects. Math anxiety can be defined as a feeling of tension or fear that interferes with operating and manipulating mathematical facts, numbers or solving mathematical problems. It manifests itself as a negative emotional response to the contents of mathematics. Highly anxious people avoid mathematics content and have negative attitudes towards mathematics, as well as a negative perception of their mathematical abilities. Negative math experience is a key factor in the development of math anxiety. Children are essentially motivated to overcome mathematical concepts, however negative attitudes of the environment, fear of failure, as well as comparison with peers escalate into negative feelings. Fear can appear when solving one or more mathematical concepts. Math anxiety in school-age students is considered a prerequisite stage of math anxiety in adulthood. Students with math anxiety avoid math and learn less than students who do not have math anxiety (Ashcraft, 2002). As a result, they show lower math achievement. Their poor results are due to low competence and achievement, not increased math anxiety. Therefore, it is very important to examine math anxiety on different math tasks. Students with a high level of math anxiety do not have a global deficit in math, they can be successful on one or more types of math tasks that do not cause them discomfort and fear. Although,

based on all of the above, mathematics can be experienced as a ghost and an insurmountable obstacle for most students, it can be overcome with a lot of work and effort. Both teachers and people involved in educational reforms in Serbia should contribute to this, as well as students who should understand and accept that mathematics is a subject that is learned continuously and that it is a long but rewarding process. Examining the relationship between math anxiety and achievement on the math literacy scale, Videnović and Radišić (2011) believe that math anxiety includes feeling pressure, inadequacy, and anxiety during solving tasks that involve manipulating numbers and solving math problems, citing PISA test data.

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