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Regional household poverty and mobility analysis – a transition probability approach[2]

Introduction

In the last few decades, economics literature has paid more attention to the issues of poverty[3]. Through websites and reports, various international institutions, including the World Bank and the European Commission, constantly compute, analyse and provide a large number of indices that enable a wider understanding of poverty (Narayan et al., 2018; World Bank, 2005; 2018; Eurobarometer, 2010). Such in-depth diagnoses are important because of the undeniable connection between poverty, social problems and their implications on the one hand, and economic growth at both regional and national levels on the other. A high likelihood of becoming poor could lead to serious problems like social exclusion (see Bieńkuńska, 2013), as well as affecting intergenerational mobility and poverty transfer from parents to children (see Corak, 2013).

The aim of this study was to estimate and analyse the transition-probability matrices on the NUTS-2 territorial-disaggregation level, emphasising the poverty class. Such estimation and analysis can help to identify which regions have the highest probability of remaining poor or of becoming poorer. The analysis used data from the Household Budget Survey (HBS), specifically the panel subsample of non-identifiable microdata concerning expenditures per capita and per equivalent unit. In addition, the study aimed to assess class-mobility expenditures

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[3] It is worth noting that Banerjee, Duflo and Kremer were awarded the 2019 Nobel Prize in Economics for their contribution to poverty reduction.

using Shorrocks' and Bartholomew's mobility indices and to assess inequalities within regions using the Gini index.

Literature overview and method description

Poverty can be defined in many ways, but it is a phenomenon that affects nearly every society as a complex, multidimensional problem. The pertinent literature widely discusses basic questions regarding poverty, including how to define it, who is really poor and whether the income criterion is sufficient to describe it. The World Bank (2005, p. 9) defines poverty as a "pronounced deprivation in well-being". The present study uses the monetary approach to that definition; therefore, the registered level of expenditures per capita (or per equivalent unit) to determine whether or not someone is poor[4]. Poverty analysis uses a wide range of methods, depending on which aspect of poverty is being analysed: its incidence, depth, intensity and severity. A sufficient overview of these methods is far beyond the scope of this paper, but can be found in Panek (2014), World Bank (2005), Haughton and Khandker (2009) and Atkinson (1987; 2019).

The present study used the classic Markov chain approach, focusing mainly on the matrix of transition probabilities, which can be defined as follows (Rey, 2014; Quah, 1995a; 1995b; Kordos, 1973; Edigarian et al., 2013):

$$P_{t/t+1} = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1k} \\ p_{21} & p_{22} & \cdots & p_{2k} \\ \vdots & \vdots & \vdots & \vdots \\ p_{k1} & p_{k2} & \cdots & p_{kk} \end{bmatrix} \tag{1}$$

where p_{ij} is a non-negative value that can be interpreted as the probability of an individual moving from state (class) *i* at time *t* to state *j* at time *t+1*. The sum of the probabilities in each row of the matrix is equal to 1.

The elements of the transition matrix can be estimated using the following unbiased, consistent maximum likelihood estimator (Rey, 2014; Kordos, 1973):

$$\hat{p}_{ij}(t) = \frac{n_{ij}}{n_{i,t}} \tag{2}$$

where n_{ij} is the number of observed transitions between states *i* and *j* during the period analysed. $n_{i,t}$ is the number of individuals who are in state *i* at time *t*.

[4] The other approach refers to the ability to obtain specific types of goods, including food, education and healthcare (World Bank, 2005; Sen, 1981).

The presented transition-probability matrix can be used to assess the general scale of mobility in terms of distinct states (classes)[5] by computing Shorrocks' mobility index (Shorrocks, 1978; Panek, 2014):

$$\widehat{M}(P) = \frac{k - trace(P)}{k - 1} \tag{3}$$

where k is the number of distinct classes in matrix P. This index is equal to 0 when perfect immobility occurs. In that case, all the diagonal elements of matrix P are equal to 1, so the trace is equal to the number of analysed classes. The higher the value of Shorrocks' index, the higher the mobility.

The Shorrocks' mobility index however takes into account only probabilities on the main diagonal of the transition matrix. The index ignores the distance that was travelled by a particular member of a household. That is, there is no difference between individuals that moved between the income classes 1 and 2 or classes 1 and 4. Obviously the second example implies greater mobility. In order to measure the scale of the distance that is travelled by the movers, the Bartholomew's Index of Social Mobility could be used (Bartholomew, 1973):

$$\hat{B}(P) = \frac{1}{k}\sum_{i=1}^{k}\sum_{j=1}^{k} p_{ij}|i - j| \tag{4}$$

This index is also equal to 0 when perfect immobility occurs. The higher the value of Bartholomew's index, the higher the mobility.

Application of the Markov chain framework to income analysis is not new. In addition, studies concerning Poland have been conducted by, for example, Kordos (1973) and Czajkowski (2009) and have focused mainly on forecasting income distributions[6]. One example of a poverty study can be found in Panek (2014), but that study does not include the regional aspect. Edigarian, Kościelniak and Trojak (2013) used transition-probability matrices to analyse regional differences in economic development using some synthetic measure as a base[7].

The present study used the Gini index to highlight the differences in expenditure inequalities among Poland's regions. One common way to define the index is as follows (Kot, 2000; Jędrzejczak, 2011):

[5] The exact schema of expenditure classes will be presented in the next section.

[6] The forecasting formula requires holding some key assumptions, including, time homogeneity (meaning that transition probabilities are constant over time). An overview of the assumptions and properties of Markov chains can be found in, for example, Shorrocks (1976) or Edigarian, Kościelniak and Trojak (2013).

[7] Of course, Markov chains are widely used in other economic fields, including modelling excepted time to default (Górajski et al., 2016).

$$G = \frac{\Delta}{2\mu}, \quad \Delta = E\left|Y_i - Y_j\right| \tag{5}$$

where Δ is the mean absolute difference between all pairs of incomes (expenditures) and μ stands for mean income (expenditures). The value of the Gini index is equal to 0 in the case of an equally distributed category of analysis (i.e. income). The higher the value of the Gini index, the more unequal is the distribution (the upper limit for the value is 1).

Data used

The present study used non-identifiable microdata from the HBS. The emphasis was on the regional aspect. The period of analysis was two years: 2015 and 2016. This is a relatively short period, but the design of the HBS enables the observation of the same sub-sample of households through both years[8]. As previously mentioned, the study used two categories, expenditures per capita and expenditures per equivalent unit, both of which were expressed in real terms using constant prices from the first quarter of 2015[9]. In poverty analysis, another common practice is to use household income. The literature provides some evidence in favour of using expenditures instead of income, especially in developed countries. This is so for two reasons: the income level might be underestimated by the respondents in certain cases, including attempts to hide illegal income, while fluctuations are more pronounced in income than in expenditure (World Bank, 2005)[10].

One problem with using HBS data to measure poverty is that households differ in size and demographic structure. To accommodate these differences, in the present study, expenditures were computed per equivalent unit using the modified Organisation for Economic Co-operation and Development (OECD) equivalence scale. The scale weights were as follows (CSO, 2017, p. 35):

- 1 is assigned for the first adult in the household,
- 0.5 is assigned for every other adult in the household,
- 0.3 is assigned for each child, defined as a person under 14 years old.

[8] Each year of the HBS data uses two overlapping sub-samples. One of these was also used in the previous year, and the other one will be used in the next year. The present study used one sub-sample, which enabled observation of year-to-year changes in expenditures for the same set of households. Detailed information about the HBS design can be found in the Central Statistical Office (CSO) (2016; 2017).

[9] Inflation will be taken into account using a quarterly consumer price index (CPI) deflator (CSO, 2019, http) due to available database information regarding the monthly rotation of households (CSO, 2016; 2017).

[10] Obviously, using expenditures has some disadvantages, including the possibility that some households are hiding either true luxuries or spending on illicit items. This could potentially lead to bias in the estimates of transition probabilities.

For the purpose of the present study, expenditure classes were set using the schema presented below, with the poverty line being 50% of the mean expenditures per capita or per equivalent unit in a particular region and in a particular year[11].

- Class 1 – below or equal to 50% of mean expenditures.
- Class 2 – 50–75% of mean expenditures.
- Class 3 – 75–100% of mean expenditures.
- Class 4 – 100–125% of mean expenditures.
- Class 5 – 125–150% of mean expenditures.
- Class 6 – more than 150% of mean expenditures.

Because there is a significant diversification in expenditure categories among regions, the study used average expenditures computed separately for each region rather than one mean defined at the national level. Figure 1 shows this diversification, as averaged, for 2015 and 2016.

The highest values for both per capita and per equivalent unit expenditures were noted for Mazowieckie. The mean value for the per capita expenditures was 119.6% of the average level for Poland, while for the per equivalent unit expenditures, it was 119.4% (PLN 1320 and PLN 2028, respectively). Relatively high values were also noted for Łódzkie, Dolnośląskie and Śląskie.

The lowest values for the categories analysed were noted for Podkarpackie. For expenditures per capita, it was 79.9% of the average level for Poland, and for expenditures per equivalent unit, it was 83.4% (PLN 882 and PLN 1417, respectively). Relatively low values were also noted for Świętokrzyskie, Warmińsko-Mazurskie and Wielkopolskie.

In relative terms, there were no significant differences between expenditures per capita and expenditures per equivalent unit[12], although in absolute values the average of the latter was notably higher for all regions[13].

RESULTS

The following includes a short analysis of the estimated-transition probabilities of the expenditure classes during 2015 and 2016 for Poland's regions (NUTS-2 level). Table 2 presents all transition matrices. For each region and each category of expenditures, the elements of the diagonals of the matrices were interpreted as the probabilities that a particular member of a household would remain in the same

[11] Expenditures in the following schema refer to expenditures per capita and expenditures per equivalence unit (at constant prices) for selected sub-sample.

[12] In the sense of share for average expenditures in particular regions in the average expenditures for the country as a whole.

[13] This is a result of dividing expenditures by the values from the equivalence scale. The values from the scale are usually lower than the number of household members.

expenditure class for one year[14]. The overall between-class mobility, as measured using Shorrocks' mobility index, is presented in Table 1. Additionally, Bartholomew's mobility index was also computed.

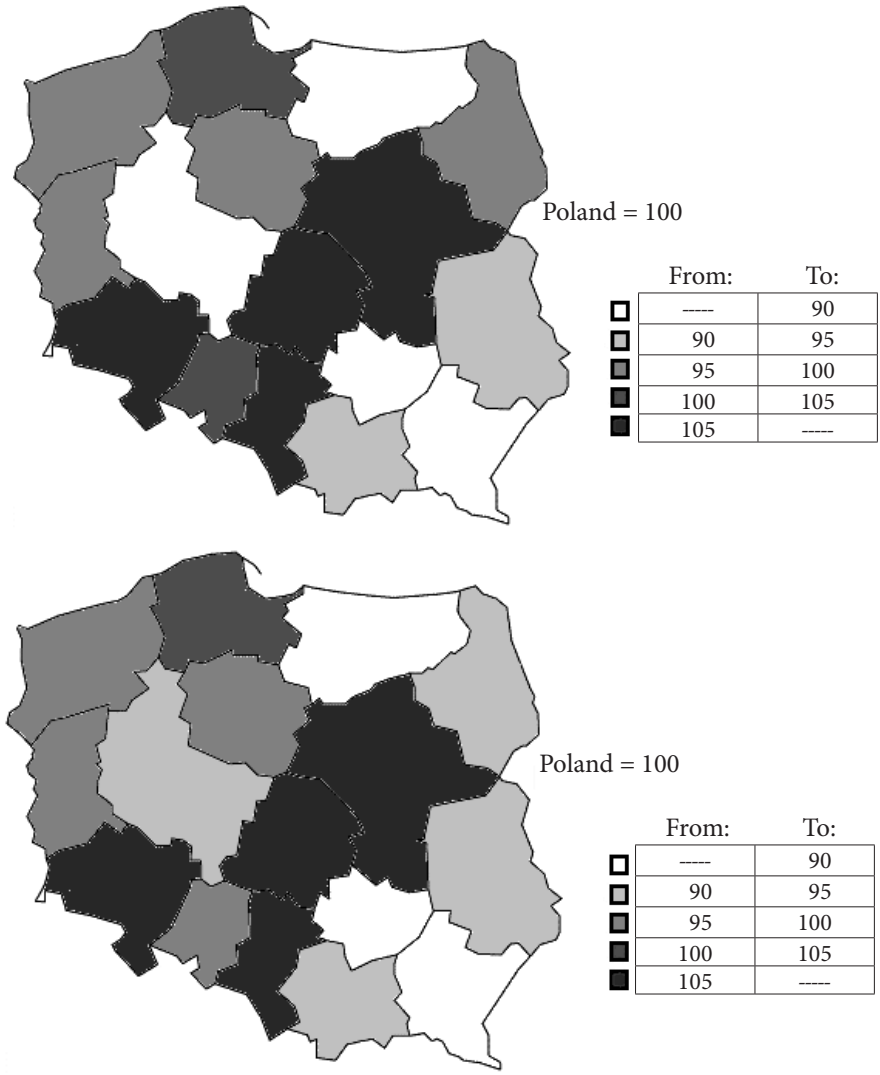

Figure 1. Regional diversification of real expenditures per capita (upper) and per equivalent (lower) units, based on the averages for 2015 and 2016 (relative values)

Source: own study based on HBS data.

[14] In fact, as suggested by Kot (2004, pp. 253–255), the data was also weighted by the number of household members. As a result, there was a shift from household distribution to individual distribution.

Table 1. Estimates of Shorrocks' and Bartholomew's Mobility Indices

Voivodeship	Expenditures per capita				Expenditures per equivalent unit			
	Shorrocks' mobility index	Bartholomew's mobility index			Shorrocks' mobility index	Bartholomew's mobility index		
		downward	upward	overall		downward	upward	overall
Dolnośląskie	0.6890	0.4600	0.3877	0.8477	0.7186	0.4689	0.4197	0.8886
Kujawsko-Pomorskie	0.7251	0.4725	0.3947	0.8671	0.7723	0.5317	0.4136	0.9453
Lubelskie	0.6766	0.4606	0.3894	0.8500	0.6959	0.4594	0.4003	0.8597
Lubuskie	0.7196	0.4678	0.4405	0.9083	0.7613	0.4836	0.4732	0.9568
Łódzkie	0.7269	0.4851	0.4044	0.8894	0.7384	0.5028	0.3954	0.8982
Małopolskie	0.7371	0.4647	0.4154	0.8801	0.7767	0.4971	0.4608	0.9579
Mazowieckie	0.6869	0.4822	0.3635	0.8457	0.6957	0.4810	0.3682	0.8492
Opolskie	0.6897	0.5334	0.3201	0.8535	0.6581	0.5729	0.3015	0.8745
Podkarpackie	0.6539	0.4338	0.3540	0.7878	0.7010	0.4410	0.3580	0.7989
Podlaskie	0.6709	0.5177	0.3626	0.8802	0.7337	0.5354	0.4029	0.9383
Pomorskie	0.6717	0.4961	0.3394	0.8355	0.7345	0.5046	0.3856	0.8902
Śląskie	0.7024	0.4868	0.3785	0.8652	0.7383	0.5079	0.3881	0.8960
Świętokrzyskie	0.7044	0.5587	0.3206	0.8793	0.7303	0.5503	0.3283	0.8786
Warmińsko-Mazurskie	0.7203	0.4823	0.3967	0.8790	0.7372	0.5101	0.4126	0.9227
Wielkopolskie	0.7096	0.4854	0.3770	0.8624	0.7532	0.5031	0.4147	0.9179
Zachodniopomorskie	0.6574	0.4107	0.3638	0.7745	0.6887	0.4107	0.4007	0.8114

Note: The lower the values of all indices, the lower mobility. Downward and upward refers to mobility, respectively towards and against poverty.

Source: own study based on HBS data.

Table 2. Estimates of Regional Transition-Probability Matrices

Voivodeship	Class #	Expenditures per capita						Expenditures per equivalent unit					
		1	2	3	4	5	6	1	2	3	4	5	6
Dolnośląskie	1	0.5634	0.3061	0.1049	0.0236	0.0020	0.0000	0.5136	0.3260	0.1230	0.0292	0.0052	0.0029
	2	0.1632	0.4856	0.2409	0.0654	0.0180	0.0269	0.1295	0.4318	0.2838	0.0933	0.0406	0.0212
	3	0.0295	0.2976	0.3639	0.1685	0.0819	0.0587	0.0504	0.2874	0.3802	0.1574	0.0699	0.0548
	4	0.0128	0.1273	0.2845	0.2662	0.1709	0.1384	0.0034	0.1487	0.2274	0.3002	0.1634	0.1568
	5	0.0000	0.1120	0.1696	0.2434	0.2327	0.2423	0.0000	0.0618	0.1506	0.3631	0.2168	0.2078
	6	0.0076	0.0302	0.0665	0.1331	0.1195	0.6432	0.0000	0.0552	0.0915	0.1266	0.1625	0.5643
Voivodeship	Class #	1	2	3	4	5	6	1	2	3	4	5	6
Kujawsko--Pomorskie	1	0.4714	0.4188	0.0964	0.0000	0.0076	0.0058	0.4215	0.3871	0.1659	0.0220	0.0000	0.0036
	2	0.1061	0.4998	0.2783	0.0417	0.0169	0.0571	0.1022	0.4858	0.3082	0.0555	0.0265	0.0219
	3	0.0401	0.3490	0.3232	0.2121	0.0435	0.0321	0.0140	0.2923	0.3465	0.2480	0.0353	0.0640
	4	0.0381	0.0968	0.2256	0.2753	0.2574	0.1068	0.0147	0.1150	0.3593	0.1876	0.1943	0.1291
	5	0.0000	0.0355	0.2826	0.2155	0.2766	0.1898	0.0052	0.1287	0.2181	0.2727	0.2461	0.1291
	6	0.0029	0.0172	0.0864	0.1709	0.1944	0.5282	0.0000	0.0000	0.1323	0.2050	0.2117	0.4511
Voivodeship	Class #	1	2	3	4	5	6	1	2	3	4	5	6
Lubelskie	1	0.5573	0.3709	0.0466	0.0118	0.0113	0.0021	0.5645	0.3553	0.0576	0.0226	0.0000	0.0000
	2	0.1826	0.4731	0.1867	0.0887	0.0368	0.0322	0.2012	0.4424	0.2269	0.0631	0.0481	0.0182
	3	0.0795	0.2071	0.3959	0.1241	0.1112	0.0823	0.0644	0.2427	0.4176	0.1323	0.0977	0.0454
	4	0.0534	0.0806	0.3041	0.2958	0.1462	0.1199	0.0408	0.1047	0.2177	0.3457	0.0990	0.1922
	5	0.0000	0.1006	0.1384	0.2938	0.2688	0.1984	0.0048	0.0470	0.1488	0.2374	0.2158	0.3462
	6	0.0024	0.0144	0.0953	0.1000	0.1617	0.6261	0.0000	0.0573	0.1025	0.0966	0.2090	0.5346
Voivodeship	Class #	1	2	3	4	5	6	1	2	3	4	5	6
Lubuskie	1	0.5568	0.2198	0.1359	0.0640	0.0235	0.0000	0.3999	0.3763	0.1756	0.0246	0.0000	0.0236
	2	0.1227	0.4799	0.2714	0.0660	0.0377	0.0223	0.0999	0.4614	0.2510	0.1166	0.0453	0.0259
	3	0.0254	0.3587	0.3073	0.1826	0.0543	0.0718	0.0000	0.2929	0.3881	0.1668	0.1282	0.0239
	4	0.0000	0.0449	0.2989	0.3091	0.1958	0.1513	0.0000	0.1125	0.3552	0.2476	0.1378	0.1469
	5	0.0000	0.0737	0.2669	0.1676	0.2374	0.2544	0.0000	0.0718	0.1897	0.2805	0.1881	0.2698
	6	0.0271	0.0063	0.0827	0.1825	0.1897	0.5117	0.0000	0.0340	0.1397	0.1799	0.1382	0.5083

Voivodeship	Class #	1	2	3	4	5	6	1	2	3	4	5	6
Łódzkie	1	0.5150	0.3972	0.0465	0.0158	0.0023	0.0232	0.4999	0.4015	0.0697	0.0092	0.0052	0.0145
	2	0.1861	0.4763	0.2543	0.0613	0.0149	0.0071	0.1461	0.4492	0.3032	0.0596	0.0088	0.0332
	3	0.0212	0.2662	0.3551	0.2336	0.0653	0.0586	0.0150	0.2758	0.4054	0.1864	0.0719	0.0455
	4	0.0034	0.1312	0.2746	0.2748	0.1497	0.1663	0.0134	0.1300	0.3260	0.2055	0.1778	0.1472
	5	0.0212	0.1477	0.1017	0.2705	0.1677	0.2912	0.0041	0.0801	0.1983	0.3630	0.1642	0.1903
	6	0.0238	0.0123	0.0836	0.1438	0.1600	0.5764	0.0258	0.0349	0.0845	0.1294	0.1414	0.5840
Voivodeship	Class #	1	2	3	4	5	6	1	2	3	4	5	6
Małopolskie	1	0.5639	0.3186	0.0407	0.0718	0.0050	0.0000	0.3380	0.4466	0.1488	0.0379	0.0287	0.0000
	2	0.1469	0.4618	0.2973	0.0587	0.0197	0.0157	0.1365	0.5133	0.2170	0.0695	0.0357	0.0280
	3	0.0328	0.3080	0.2941	0.2393	0.0529	0.0730	0.0487	0.2641	0.3401	0.2129	0.0808	0.0533
	4	0.0257	0.1331	0.2934	0.2143	0.2135	0.1200	0.0129	0.1488	0.2948	0.2264	0.2140	0.1031
	5	0.0032	0.0989	0.2043	0.1965	0.1943	0.3027	0.0030	0.1068	0.1994	0.1948	0.2337	0.2623
	6	0.0088	0.0141	0.0534	0.1180	0.2198	0.5859	0.0026	0.0281	0.0581	0.1818	0.2642	0.4652
Voivodeship	Class #	1	2	3	4	5	6	1	2	3	4	5	6
Mazowieckie	1	0.6676	0.2559	0.0527	0.0170	0.0067	0.0000	0.5782	0.3174	0.0812	0.0104	0.0090	0.0039
	2	0.2247	0.4945	0.1733	0.0663	0.0274	0.0138	0.1830	0.5143	0.2002	0.0649	0.0243	0.0133
	3	0.0819	0.2384	0.3497	0.2026	0.0844	0.0429	0.0670	0.2843	0.3771	0.1554	0.0614	0.0548
	4	0.0306	0.1246	0.3095	0.1930	0.1478	0.1945	0.0218	0.1315	0.2662	0.2755	0.1650	0.1400
	5	0.0076	0.0677	0.2024	0.2278	0.2329	0.2616	0.0180	0.0508	0.2300	0.2210	0.1813	0.2989
	6	0.0069	0.0335	0.0603	0.1289	0.1425	0.6279	0.0020	0.0275	0.0758	0.1376	0.1619	0.5951
Voivodeship	Class #	1	2	3	4	5	6	1	2	3	4	5	6
Opolskie	1	0.6826	0.1750	0.1284	0.0000	0.0140	0.0000	0.7059	0.1309	0.0598	0.1034	0.0000	0.0000
	2	0.2185	0.4726	0.2171	0.0590	0.0090	0.0238	0.1515	0.4805	0.2254	0.0829	0.0431	0.0166
	3	0.0506	0.4590	0.2480	0.0927	0.0673	0.0824	0.0466	0.2559	0.5298	0.1020	0.0375	0.0282
	4	0.0309	0.1182	0.3281	0.3412	0.0877	0.0939	0.0398	0.1952	0.3731	0.2369	0.0368	0.1182
	5	0.0000	0.1130	0.1856	0.1630	0.3129	0.2256	0.0000	0.0714	0.2890	0.3761	0.1367	0.1268
	6	0.0000	0.0298	0.0931	0.1099	0.2728	0.4944	0.0000	0.0524	0.1400	0.0681	0.1200	0.6195

Voivodeship	Class #	1	2	3	4	5	6	1	2	3	4	5	6
Podkarpackie	1	0.7302	0.2220	0.0314	0.0163	0.0000	0.0000	0.5783	0.3357	0.0685	0.0176	0.0000	0.0000
	2	0.1245	0.4843	0.2741	0.0922	0.0194	0.0055	0.1670	0.4944	0.2426	0.0630	0.0210	0.0119
	3	0.0465	0.2734	0.3817	0.1850	0.0867	0.0267	0.0299	0.2933	0.3588	0.2131	0.0738	0.0311
	4	0.0157	0.1349	0.2060	0.3116	0.1629	0.1689	0.0153	0.1626	0.2735	0.2392	0.2240	0.0853
	5	0.0000	0.0917	0.1863	0.2028	0.2068	0.3124	0.0000	0.0429	0.1156	0.3114	0.2355	0.2946
	6	0.0000	0.0336	0.0690	0.1156	0.1657	0.6161	0.0000	0.0290	0.0781	0.1554	0.1486	0.5889
Voivodeship	Class #	1	2	3	4	5	6	1	2	3	4	5	6
Podlaskie	1	0.5765	0.2751	0.1147	0.0337	0.0000	0.0000	0.4153	0.3889	0.1540	0.0000	0.0332	0.0086
	2	0.1698	0.5286	0.2157	0.0319	0.0319	0.0222	0.1881	0.4666	0.2797	0.0425	0.0100	0.0131
	3	0.0692	0.3142	0.3611	0.1326	0.0422	0.0806	0.0521	0.2965	0.3535	0.1518	0.0434	0.1028
	4	0.0262	0.1205	0.2922	0.2852	0.1060	0.1700	0.0224	0.1574	0.3519	0.2766	0.1119	0.0798
	5	0.0311	0.1011	0.1922	0.2351	0.2397	0.2009	0.0000	0.0638	0.2331	0.1556	0.2683	0.2791
	6	0.0253	0.0555	0.0787	0.0538	0.1323	0.6542	0.0000	0.0973	0.1234	0.0894	0.1385	0.5514
Voivodeship	Class #	1	2	3	4	5	6	1	2	3	4	5	6
Pomorskie	1	0.7185	0.2068	0.0462	0.0232	0.0000	0.0053	0.5633	0.3195	0.0804	0.0247	0.0050	0.0071
	2	0.2209	0.4722	0.1852	0.0749	0.0156	0.0312	0.1981	0.4639	0.2165	0.0746	0.0181	0.0287
	3	0.0194	0.2974	0.3905	0.1628	0.0766	0.0534	0.0198	0.3257	0.3388	0.2093	0.0702	0.0362
	4	0.0198	0.2383	0.2190	0.2434	0.1292	0.1503	0.0249	0.1139	0.3427	0.2390	0.1269	0.1527
	5	0.0000	0.0690	0.1998	0.3068	0.1959	0.2285	0.0089	0.0301	0.2103	0.2573	0.2152	0.2782
	6	0.0111	0.0188	0.0602	0.1512	0.1375	0.6211	0.0118	0.0503	0.0779	0.1687	0.1839	0.5073
Voivodeship	Class #	1	2	3	4	5	6	1	2	3	4	5	6
Śląskie	1	0.5766	0.3193	0.0646	0.0333	0.0050	0.0012	0.4612	0.4309	0.0896	0.0182	0.0000	0.0000
	2	0.1222	0.5486	0.2383	0.0627	0.0145	0.0138	0.1417	0.5116	0.2320	0.0722	0.0272	0.0153
	3	0.0494	0.2867	0.3816	0.1450	0.0804	0.0569	0.0437	0.2351	0.4257	0.1584	0.0780	0.0590
	4	0.0145	0.1308	0.2778	0.2596	0.1308	0.1866	0.0035	0.1040	0.4078	0.2261	0.1374	0.1211
	5	0.0051	0.0432	0.1904	0.3130	0.1948	0.2534	0.0085	0.0727	0.1713	0.2682	0.2054	0.2739
	6	0.0078	0.0497	0.0924	0.1477	0.1757	0.5267	0.0110	0.0301	0.1209	0.1963	0.1630	0.4787

Voivodeship	Class #	1	2	3	4	5	6	1	2	3	4	5	6
Świętokrzyskie	1	0.6499	0.2660	0.0634	0.0052	0.0154	0.0000	0.5973	0.3010	0.0792	0.0148	0.0078	0.0000
	2	0.1846	0.4893	0.2554	0.0373	0.0151	0.0183	0.2435	0.3540	0.2953	0.0700	0.0168	0.0204
	3	0.0580	0.2561	0.4554	0.1585	0.0185	0.0535	0.0199	0.2833	0.4032	0.2177	0.0407	0.0353
	4	0.0835	0.1143	0.3658	0.1276	0.1714	0.1374	0.0101	0.2008	0.3087	0.2905	0.1397	0.0502
	5	0.0000	0.0918	0.2692	0.2404	0.1959	0.2027	0.0000	0.1158	0.1103	0.3260	0.2254	0.2224
	6	0.0089	0.0277	0.0913	0.1551	0.1569	0.5600	0.0109	0.0337	0.0689	0.2961	0.1121	0.4783
Voivodeship	Class #	1	2	3	4	5	6	1	2	3	4	5	6
Warmińsko--Mazurskie	1	0.6039	0.2888	0.0562	0.0462	0.0000	0.0049	0.5724	0.2012	0.2049	0.0154	0.0061	0.0000
	2	0.2396	0.4930	0.1608	0.0780	0.0185	0.0101	0.1784	0.4522	0.2397	0.0733	0.0489	0.0074
	3	0.0426	0.3768	0.2598	0.1907	0.0193	0.1107	0.0683	0.2552	0.3659	0.1676	0.0917	0.0512
	4	0.0085	0.0476	0.3366	0.2395	0.1750	0.1928	0.0000	0.1506	0.3265	0.2344	0.1620	0.1266
	5	0.0358	0.0914	0.1276	0.2753	0.1885	0.2814	0.0000	0.0233	0.2670	0.2661	0.1318	0.3118
	6	0.0000	0.0180	0.0984	0.1497	0.1201	0.6139	0.0208	0.0183	0.1511	0.1093	0.1433	0.5572
Voivodeship	Class #	1	2	3	4	5	6	1	2	3	4	5	6
Wielkopolskie	1	0.5704	0.3370	0.0586	0.0169	0.0084	0.0088	0.4398	0.4142	0.0992	0.0120	0.0264	0.0084
	2	0.1868	0.4417	0.2507	0.0998	0.0076	0.0135	0.1826	0.4643	0.2323	0.0808	0.0362	0.0037
	3	0.0331	0.2795	0.3442	0.2467	0.0665	0.0300	0.0355	0.2804	0.3494	0.2220	0.0942	0.0185
	4	0.0294	0.0931	0.2667	0.2993	0.1980	0.1134	0.0077	0.0865	0.3083	0.2637	0.1922	0.1416
	5	0.0117	0.0952	0.2109	0.2190	0.2055	0.2578	0.0109	0.1084	0.1741	0.2844	0.1886	0.2336
	6	0.0000	0.0376	0.1015	0.1406	0.1293	0.5910	0.0060	0.0232	0.1297	0.1543	0.1585	0.5284
Voivodeship	Class #	1	2	3	4	5	6	1	2	3	4	5	6
Zachodniopomorskie	1	0.6037	0.3419	0.0329	0.0106	0.0075	0.0033	0.5712	0.2742	0.0978	0.0568	0.0000	0.0000
	2	0.0780	0.4540	0.4032	0.0217	0.0311	0.0118	0.0817	0.4230	0.4069	0.0470	0.0226	0.0189
	3	0.0315	0.2409	0.4361	0.1934	0.0739	0.0243	0.0347	0.2078	0.4135	0.2577	0.0555	0.0308
	4	0.0073	0.1067	0.2451	0.2794	0.2277	0.1338	0.0069	0.0924	0.2783	0.3436	0.2098	0.0691
	5	0.0000	0.1089	0.1534	0.2663	0.2711	0.2003	0.0000	0.0693	0.1781	0.2040	0.2379	0.3106
	6	0.0201	0.0320	0.0519	0.0904	0.1371	0.6685	0.0000	0.0593	0.0586	0.1256	0.1889	0.5675

The higher the values, the higher is the probability of transition to particular income class (being in a given income class at time *t*).

Source: own study based on HBS data.

Based on Shorrocks' index, the highest between-class mobility was noted for Małopolskie. The mobility rankings for regions varied for both variables, but Łódzkie, Kujawsko-Pomorskie and Lubuskie also could be perceived as high--mobility regions. The ranking for regions having low mobility was ambiguous: For expenditures per capita, Podkarpackie, Zachodniopomorskie and Podlaskie achieved rather low values of Shorrocks' index. For expenditures per equivalent unit, low values were noted for Opolskie (which had been in the middle of the previous ranking), Zachodniopomorskie and Mazowieckie. The general conclusion is that taking into account differences in the sizes and demographic structures of the households notably affected the overall mobility in the low and middle portions of the ranking. In this study, Shorrocks' index had values in the range 0–1.2, as the upper limit is not constant for this measure. For each region, the values of the mobility index were higher than 0.6 (midpoint of the interval), indicating that, in general, Poland's regions are rather mobile.

The very first element of the main diagonals of the transition matrices defined the probability of staying poor during one year (i.e. staying in the first expenditure class).

The following conclusions were reached regarding regional differentiation.

- For expenditures per capita, the highest probability of staying poor was noted for Opolskie, Pomorskie and Podkarpackie (approximately 0.68–0.73).
- For expenditures per equivalent unit, the highest probability was noted for Podkarpackie, Świętokrzyskie and Opolskie. It is worth remarking that there was a significant absolute difference between the first and second regions in this ranking (the probabilities were approximately 0.60 and 0.71, respectively).
- The lowest probability for expenditures per capita was noted for Kujawsko-Pomorskie, Łódzkie and Lubuskie (approximately 0.47–0.56).
- The lowest probability for expenditures per equivalent unit was noted for Małopolskie, Lubuskie and Podlaskie (approximately 0.34–0.42).

The very last element of the main diagonals of the transition matrices defined the probability of staying in the highest (sixth) expenditure class during one year. The following conclusions were reached regarding regional differentiation.

- For expenditures per capita, the highest probability of staying in "the richest" class was noted for Dolnośląskie, Podlaskie and Zachodniopomorskie (approximately from 0.64–0.67).
- For expenditures per equivalent unit, the highest probability was noted for Podkarpackie, Mazowieckie and Opolskie (approximately 0.59–0.62).
- The lowest probability for expenditures per capita was noted for Opolskie, Lubuskie and Śląskie (approximately 0.49–0.53).
- The lowest probability for expenditures per equivalent unit was noted for Kujawsko-Pomorskie, Małopolskie and Świętokrzyskie (approximately 0.45–0.48).

Taking into account the probabilities outside the main diagonals of the transition matrices, Mazowieckie had the highest probability of falling into the first expenditure class from another class for expenditures per capita and Lubelskie had the highest

probability for expenditures per equivalence unit. The lowest levels were for Zachodniopomorskie and Lubuskie (for both types of expenditures)[15].

The values of the Bartholomew's mobility index suggest that higher mobility between income classes occurs towards poverty (see Table 1, "downward" and "upward" columns). Those results hold on for every analysed region. Additional conclusions are as follows.

- The highest mobility towards poverty was noted for Opolskie, Podlaskie and Świętokrzyskie (both for expenditures per capita and per equivalent unit).
- The lowest mobility towards poverty was noted for Zachodniopomorskie and Podkarpackie (also for both types of expenditures).
- The highest mobility against poverty was noted for Lubuskie and Małopolskie.
- The lowest mobility against poverty was noted for Opolskie and Świętokrzyskie.

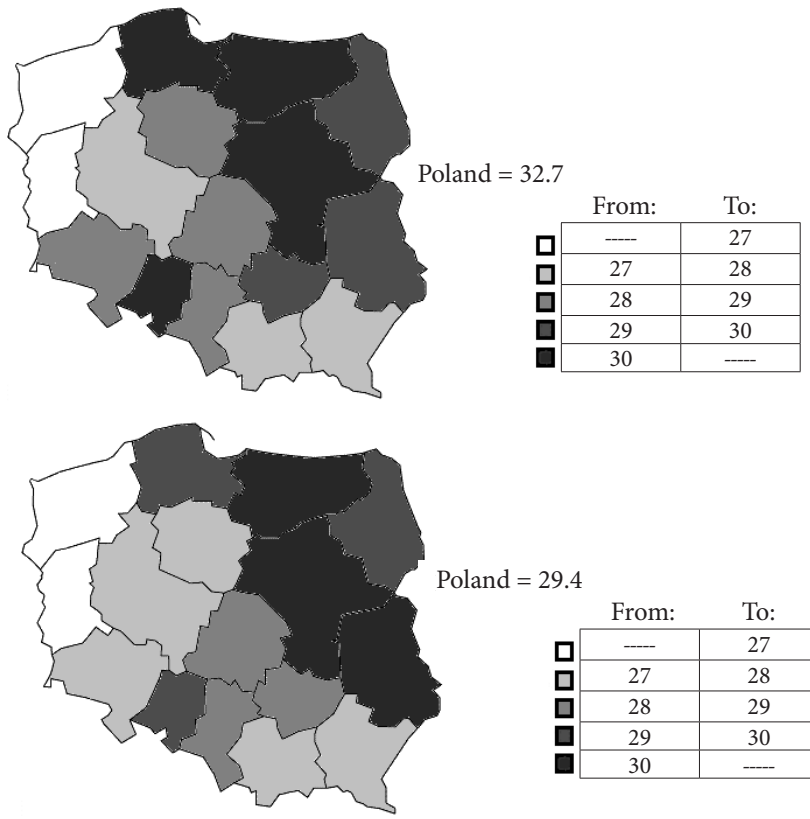

Figure 2. Regional differentiation of the Gini index for real expenditures per capita (upper) and per equivalent (lower) units, based on the averages for 2015 and 2016

Source: own study based on HBS data.

[15] Based on the probabilities from the first columns of Table 2.

Figure 2 shows the estimated values of the Gini indices. The highest expenditure inequality was found in Mazowieckie. The Gini index for that region was equal to approximately 0.35 for expenditures per capita and approximately 0.32 for expenditures per equivalent unit. A high inequality level was also identified for Warmińsko-Mazurskie.

The lowest inequality level was noted for Lubuskie in expenditures per capita (approximately 0.28) and for Zachodniopomorskie in per equivalent units (approximately 0.25). Low inequality levels were also identified for Wielkopolskie, Podkarpackie and Małopolskie.

Conclusions

Poland's regions are distinguished by significant differences in levels of average expenditures per capita and per equivalent unit as well as in their levels of transition probability. In addition, the estimates of Shorrocks' mobility index show that some differences exist in these levels among regions. The exact ranking is sensitive regarding the category of expenditures applied, but in general, Poland's regions are rather mobile. Bartholomew's index revealed that Polish regions were characterized by a greater "towards poverty" mobility (in comparison to upward mobility). This is a serious problem that can lead to social exclusion. Of course, probabilities estimated only on the basis of a single two-year sample cannot answer questions about the nature of the poverty in particular regions. These questions about the persistence of poverty are important, especially in the context of social-policy strategies, and should deepen the regional dimension of future research (see Panek, 2014)[16]. Furthermore, the Gini index values show some differences in expenditure inequality among regions, although the ranking is fairly consistent between the per capita and per equivalent unit versions of expenditures.

Except for Opolskie and Lubelskie, all regions have lower probabilities of staying poor (in the first class) regarding expenditures per equivalent unit than per capita expenditures. In addition, the Gini index of inequality was lower for equivalent unit cases. Similar results regarding inequality measurement were obtained by Kot (2004, p. 267).

The monetary approach to poverty analysis that was used in this study does not cover all the aspects of poverty itself. The literature also suggests taking into consideration the multidimensional framework for computing poverty indices. Alkire, and Foster measures, for example, make use of weighted dimensions that refer to education, health and living environment like pollution or safety (see Alkire,

[16] In addition to the question of the nature of poverty, there is also the issue of how stable in time the estimated probabilities are. This stability issue could be checked using other samples from other years and will be the subject of future research.

Apablaza, 2016). Without any doubt, the true nature of poverty cannot be explained by just one dimension.

Finally, it is worth noting that obtained results suggest that social policy should put more emphasis on the spatial diversification of poverty. The relative distance between the "poor" in Mazowieckie region and the "poor" in Świętokrzyskie region could be significant. A similar problem refers to particular individuals' chance of improving their overall situation. Thus, this implies a more contextual approach to policymaking.

BIBLIOGRAPHY

Alkire, S., Apablaza, M. (2016). Multidimensional poverty in Europe 2006–2012: Illustrating a methodology. *OPHI Working Paper, 74*, 1–22.

Atkinson, A. B. (1987). On the measurement of poverty. *Econometrica*, *55*(4), 749–764. DOI: 10.2307/1911028.

Atkinson, A. B. (2019). *Measuring poverty around the world*. New Jersey: Princeton University Press. DOI: 10.1515/9780691191898.

Bartholomew, D. J. (1973). *Stochastic Models for Social Processes*. London: J. Wiley and Sons. DOI: 10.1002/bimj.19740160813.

Bieńkuńska, A. (2013). Multidimensional poverty and social isolation in Poland. *Working Paper, 20*, 1–14.

Corak, M. (2013). Income inequality, equality of opportunity, and intergenerational mobility. *Journal of Economic Perspectives*, *27*(3), 79–102. DOI: 10.1257/jep.27.3.79.

CSO (2016). *Household budget survey in 2015*. Warsaw: Central Statistical Office.

CSO (2017). *Household budget survey in 2016*. Warsaw: Central Statistical Office.

CSO (2019). *Local data bank*. Retrieved from: https://bdl.stat.gov.pl/BDL/start (2019.6.06).

Czajkowski, A. (2009). Forecasting income distributions of households in Poland on the basis of Markov chains. *Acta Universitatis Lodziensis*, *225*, 245–256.

Edigarian, E., Kościelniak, P., Trojak, M. (2013). Analiza zróżnicowania rozwoju ekonomicznego województw i powiatów oparta na łańcuchach Markowa. In: M. Trojak, T. Tokarski (Eds.), *Statystyczna analiza przestrzennego zróżnicowania rozwoju ekonomicznego i społecznego Polski* (pp. 111–128). Kraków: Wydawnictwo Uniwersytetu Jagiellońskiego.

Eurobarometer (2010). Poverty and social exclusion. *Special Eurobarometer, 321*. Retrieved from: https://ec.europa.eu/commfrontoffice/publicopinion/archives/ebs/ebs_321_en.pdf (2019.06.06).

Haughton, J., Khandker, S. R. (2009). *Handbook on poverty and inequality*. Washington D.C.: World Bank. DOI: 10.1596/978-0-8213-7613-3.

Górajski, M., Serwa, D., Wośko, Z. (2016). Measuring expected time to default under stress conditions for corporate loans. *NBP Working Paper, 237*, 1–36.

Jędrzejczak, A. (2011). *Metody analizy rozkładów dochodów i ich koncentracji*. Łódź: Wydawnictwo Uniwersytetu Łódzkiego.

Kordos, J. (1973). *Metody analizy i prognozowania rozkładów płac i dochodów ludności*. Warszawa: Państwowe Wydawnictwo Ekonomiczne.

Kot, S. M. (2000). *Ekonometryczne modele dobrobytu*. Warszawa–Kraków: Wydawnictwo Naukowe PWN.

Kot, S. M. (2004). Rozkłady dochodów, nierówności i dobrobyt w Polsce. In: S. M. Kot, A. Malawski, A. Węgrzecki (Eds.), *Dobrobyt społeczny, nierówności i sprawiedliwość dystrybutywna* (pp. 248–280). Kraków: Wydawnictwo Akademii Ekonomicznej w Krakowie.

Narayan, A., Van der Weide, R., Cojocaru, A., Lakner, C., Redaelli, S., Mahler, D. G., Thewissen, S. (2018). *Fair progress?: Economic mobility across generations around the world. Equity and development*. Washington D.C.: World Bank. DOI: 10.1596/978-1-4648-1210-1.

Panek, T. (2014). Ubóstwo i wykluczenie społeczne. In: T. Panek (Ed.), *Statystyka społeczna* (pp. 195–239). Warszawa: Polskie Wydawnictwo Ekonomiczne.

Quah, D. T. (1995a). Aggregate and regional disaggregate fluctuations. *Centre for Economic Performance Discussion Paper No. 275*.

Quah, D. T. (1995b). Empirics for economic growth and convergence. *Centre for Economic Performance Discussion Paper No. 253*.

Rey, S. (2014). Rank-based Markov chains for regional income distributions dynamics. *Journal of Geographical Systems*, *16*(2), 115–137. DOI: 10.1007/s10109-013-0189-0.

Sen, A. K. (1981). *Poverty and famines: An essay on entitlement and deprivation*. Oxford: Clarendon Press. DOI: 10.5040/9781474220156.ch-005.

Shorrocks, A. F. (1976). Income mobility and the Markov assumption. *The Economic Journal*, *86*(343), 566–578. DOI: 10.2307/2230800.

Shorrocks, A. F. (1978). The measurement of mobility. *Econometrica*, *46*(5), 1013–1024. DOI: 10.2307/1911433.

World Bank (2005). *Introduction to poverty analysis*. World Bank Institute. Retrieved from: http://siteresources.worldbank.org/PGLP/Resources/PovertyManual.pdf (2019.06.15).

World Bank (2018). *Poverty*. Retrieved from: https://www.worldbank.org/en/topic/poverty (2019.08.18).

Summary

The main objective of this paper was to estimate and analyse transition-probability matrices for all 16 of Poland's NUTS-2 level regions (voivodeship level). The analysis is conducted in terms of the transitions among six expenditure classes (per capita and per equivalent unit), focusing on poverty classes. The period of analysis was two years: 2015 and 2016. The basic aim was to identify both those regions in which the probability of staying in poverty was the highest and the general level of mobility among expenditure classes. The study uses a two-year panel sub-sample of unidentified unit data from the Central Statistical Office (CSO), specifically the data concerning household budget surveys. To account for differences in household size and demographic structure, the study used expenditures per capita and expenditures per equivalent unit simultaneously. To estimate the elements of the transition matrices, a classic maximum-likelihood estimator was used. The analysis used Shorrocks' and Bartholomew's mobility indices to assess the general mobility level and the Gini index to assess the inequality level.

The results show that the one-year probability of staying in the same poverty class varies among regions and is lower for expenditures per equivalent units. The highest probabilities were

identified in Podkarpackie (expenditures per capita) and Opolskie (expenditures per equivalent unit), and the lowest probabilities in Kujawsko-Pomorskie (expenditures per capita) and Małopolskie (expenditures per equivalent unit). The highest level of general mobility was noted in Małopolskie, for both categories of expenditures.

Keywords: poverty, transition probability, Markov chains, mobility, inequality, regional analysis.

Regionalna analiza ubóstwa i mobilności gospodarstw domowych – podejście oparte na prawdopodobieństwie przejścia

Streszczenie

Głównym celem niniejszego artykułu była próba estymacji i analizy macierzy prawdopodobieństw przejścia, określonej dla wszystkich szesnastu regionów Polski (województwa, poziom NUTS-2). Analiza została przeprowadzona pod kątem przejść pomiędzy sześcioma klasami wydatków (w ujęciu *per capita* oraz na jednostkę ekwiwalentną), ze szczególnym uwzględnieniem sfery ubóstwa. Okres analizy obejmował dwa lata: rok 2015 oraz 2016. Podstawowe pytanie dotyczyło tego, w których regionach prawdopodobieństwo pozostawania w biedzie jest największe oraz jaki jest ogólny poziom mobilności pomiędzy klasami wydatków. Do badania została wykorzystana dwuletnia podpróba panelowa oparta o nieidentyfikowalne dane jednostkowe Głównego Urzędu Statystycznego, pochodzące z badania budżetów gospodarstw domowych. W badaniu wykorzystano zarówno wydatki *per capita*, jak i wydatki na jednostkę ekwiwalentną, aby wziąć pod uwagę różnice w wielkości i strukturze demograficznej gospodarstw domowych. Elementy macierzy przejścia były szacowane za pomocą klasycznego estymatora największej wiarygodności. Analiza została uzupełniona ogólną oceną mobilności za pomocą indeksów mobilności Shorrocksa i Bartholomewa oraz oceną poziomu nierówności wydatków za pomocą indeksu Giniego.

Wyniki pokazały, że roczne prawdopodobieństwa pozostawania w biedzie różnią się w zależności od regionu i są mniejsze dla wydatków na jednostkę ekwiwalentną. Największe prawdopodobieństwo zaobserwowano dla województwa podkarpackiego (wydatki *per capita*) oraz opolskiego (wydatki na jednostkę ekwiwalentną). Najniższym prawdopodobieństwem odznaczało się województwo kujawsko-pomorskie (wydatki *per capita*) oraz małopolskie (wydatki na jednostkę ekwiwalentną). Najwyższym ogólnym poziomem mobilności charakteryzowało się województwo małopolskie (dla obu kategorii wydatków).

Słowa kluczowe: ubóstwo, prawdopodobieństwa przejścia, łańcuchy Markowa, mobilność, nierówności, analiza regionalna.

JEL: I32, C10, R10.