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Reflections on the Formation of a Child's Mathematical Culture at the Beginning of School Maturity

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Abstract

The authors of the article discuss the issue of mathematical culture concerning the education of the youngest participants of the educational system – children, in the course of preschool and school education in grades 1–3. The authors also refer to the problem of developing the mathematical culture of the school pupil, in the perspective of the transformation of the Polish educational system, challenges connected with modern education and its transformation on the national area.

Keywords: mathematical culture, preschool education, child, mathematics

Introduction

Often, in the context of teaching mathematics, its usefulness is questioned. Students, as well as teachers, ask questions: What will it be helpful? Will I use this knowledge? Which contents of mathematics education are unnecessary in general education? These questions prove that we forget about a more critical aspect of teaching mathematics – its culture-creating dimension. The authors of "The Queen without a Nobel Prize", among others, draw attention to this aspect: The teaching of mathematics is looked at by many people primarily in terms of practical usefulness, forgetting certain other aspects, such as general culture. Of course, we do not use various mathematical constructions and objects directly in our everyday life, but often we do not even know that we are using them or making use of their properties. (…) During maths lessons, apart from specific tools and methods, young people should learn logical thinking and conclude data, which is deductive thinking. It is what solving geometry tasks, or equations with logarithms and sines teaches. They teach the use of data, force logical thinking on abstract examples (Ciesielski, Pogoda, 2013, p. 44).

In the presented reflection, we attempt to demonstrate the importance of one of the essential mathematical competencies shaped already in the preschool period (spatial orientation) for the development of the student's mathematical culture.

The concept of mathematical culture

Duda (1990, p. 27)considers mathematics as a particular element of human culture, equipping him with the ability to practice other disciplines of knowledge, a tool for understanding the world, development of technology and civilization. He highly values the influence of Greek culture on the development of civilization and the formation of humanism of man. However, the notion of mathematical culture, like culture, is very relative and complex to define (Więsław, 1998, p. 27). While approaching the issue of searching for a modern understanding of mathematical culture, both in general and in particular, we should not overlook the historical context accompanying the genesis, development and transformations of mathematics as a field of empirical science. Although the beginnings of the history of mathematics in the Polish lands are shrouded in the darkness of the past, even at such early stages of the existence of mathematics in social life, we can see some signs of the emergence of systematic foundations of the original mathematical culture – for example measuring distances, fields and weight according to specific measures (among others: gon, stajanie, kopa, kopka). As Duda (2018 p. 26–27) points out, referring to the example of time reckoning, it should be considered at the same time that, to a significant extent, mathematics and its usefulness at the beginning of history were to be primarily qualitative rather than quantitative. Regardless of the difficulties related to the precise definition of the course of the origins of mathematics and its related culture in Poland, we can assume that it developed non-linearly and that the period of rapid, historical and developmental transformations of mathematical education and culture was the turn of the 19th and 20th centuries. It is worth noting that the described peculiar developmental leap, which occurred at that time in Polish general education, had an obvious connection with long-lasting and unceasing aspirations for independence. These aspirations in the educational sphere were expressed in publishing activities resulting in an increased number of textbooks for many school subjects, including mathematics, in Polish (Duda, 2018, p. 176). The fluctuations occurring in the history of Polish mathematical education, especially taking into account the above-described variability happening over the centuries, even to this day, raise questions about the current understanding of the essence of mathematical culture Is mathematical culture the culture of a specific professional group – e.g. teachers or theoretical mathematicians? Alternatively, maybe it is a multidimensional state of mind of people – who think in a programme-oriented, consistent, strict way? (Makiewicz, 2010, p. 23). Is mathematical culture an attribute of adults, or is it possible to shape it at preschool or younger school age? The category of mathematical culture derives from Kurina (1991, p. 30), who undertook to define the phenomenon covering:

1. Gaining mathematical proficiency.
2. Understanding the continuous transition in the different disciplines of mathematics between mathematics -science and mathematics -teaching.
3. Understanding the language of mathematics.
4. Being able to choose appropriate methods when solving tasks.
5. Having a good geometrical imagination.
6. Mastering the technique of calculation.
7. Mastering the ability to carry out proofs.
8. Mastering the ability to introduce concepts.
9. Being able to practice mathematical creativity to some extent.
10. Perception of the beauty of mathematics.

In the context of the early formation of mathematical culture, the presented "decalogue" defines forms of activity that are too distant from children's activities (e.g., introducing concepts is a skill dedicated mainly to teachers). It is worth noticing that contemporary systemic problems accompanying shaping and conducting mathematical education, and therefore also building and instilling a mathematical culture in children and young people, are noted among others valuable in-depth studies devoted to the measurement of length in primary education and the potential inherent in teaching and learning with the use of learning trajectories (Sarama et al., 2021). Moreover, recent research on the mathematical culture phenomenon reveals the ambiguity of this concept, which points to the need for constant revision in this area. Holistic, coherent and universal at the same time, treatment of only two notions: mathematical culture and culture of mathematicians, is probably difficult to achieve at the current research stage. As Leone Burton points out in her research based on interviews-dialogues with mathematicians from academic circles, among the distinguishing features, key concepts enabling to understand what, in a subjective approach, is the culture of mathematics, we can mention the following terms (Burton, 2009, p. 166):

- rigour,
- beauty,
- simplicity,
- structure,
- pattern,
- power,

– conciseness,
– symmetry.

Burton (2009, p. 159) points out that the assimilation of a catalogue of these terms is part of an introduction to the mathematical community of practice. Apart from the list of terms listed by the researcher to facilitate the grasp of the specificity of mathematical culture, the participants of the study (women and men) mentioned three specific terms-distinctions, with negative overtones, related to environmental attitudes, behaviours and values, which are elements of mathematical culture. These are:

– hierarchy,
– isolation,
– competition.

The researcher points out that his considerations on mathematicians' culture and mathematical culture, based on an epistemological model, reveal not only the ambiguity of the distinctions accompanying it, but above all with the existence of a disputable aspiration to establish an objective set of concepts characterizing mathematics as an impersonal, homogeneous and objective domain (Burton, 2009, p. 160). Regarding the ambiguities indicated above, we conclude that the existing heterogeneity and fluidity of the notions connected with mathematical culture shows the vitality of this research matter and should encourage further exploration of the described issue. Taking the above circumstances into consideration, in considering the shaping of a mathematical culture of a child educated in the national educational system, we rely on the model described by Makiewicz (2013, p. 57–59). This model is based on in-depth research that identifies the conditions necessary for the practical construction and development of a student's mathematical culture. It also defines the conditions conducive to initiating and conducting this process and consists of the following, sometimes complementary zones: 1. necessary conditions and 2. conditions conducive to developing the child's mathematical culture (Makiewicz, 2013, p. 58). The zone of necessary conditions includes two components necessary for undertaking mathematical activity: using the language of mathematics and mastering basic mathematical proficiency (Makiewicz, 2013, p. 57). The zone of conditions conducive to developing a child's mathematical culture includes experiencing mathematical creativity, having good geometrical imagination, and choosing the best way to solve a mathematical elegance problem. Without a student's command of these two components, it is impossible to speak of his/her attainment of mathematical culture. In the discussed model of mathematical culture, we distinguish the following areas:

Area J (language) is concerned with understanding and interpreting mathematical text and expressing thoughts using mathematical concepts, formulae and formulas. An example of well-formed mathematical language is the title of Photo 1. The sum of sets can be understood as the result of collecting mushrooms, or the result of a logical operation.

Photo 1. ***Union of sets*** – Jan Łachiński

Area S (proficiency) includes mathematical knowledge and skills necessary for solving tasks. In a child's mathematical development, it is crucial to forming a perception of basic skills – activities developed at different education stages constitute mathematical proficiency. Such a skill is, for example, classifying objects which differ in one feature, e.g. colour, shape, size, and then ordering objects, e.g. according to their length. According to Gruszczyk-Kolczyńska (Gruszczyk-Kolczyńska, Zielińska, 2015, p. 103), such advanced operation can be mastered by a child at age 9, while it is already expected from 6-year-olds.

Area W (imagination) refers to producing and manipulating pictorial, verbal and logical ideas related to mathematical objects. It develops from an early age. In the pre-operational period – as the formation of correct spatial orientation. It is related to the propensity of imagination and spatial orientation. The related understanding of length is associated with ability. By area T (creativity), we mean creative abilities revealed in the area of mathematics understood in Guilford's (1978) and Urban's (1990, s. 99–113) categories, which occur at fluid or crystallized level (Nęcka, 2001, s. 218), including the mini-t creative work (Kaufmann, Beghetto, 2009, p. 1–12). Mini-t creativity, considered as local, subjective creativity, is a developmental activity without which there can be no every day and outstanding creativity at a later stage (Limont, 2010, p. 127). Area E (elegance) is associated with choosing the simplest, optimal, under given conditions, way to solve a mathematical problem. The authors of "Mathematics from kindergarten" refer indirectly to the notions of creativity and mathematical elegance presented above, indicating the necessity of organising the process of introducing a preschooler to mathematics in a way that is adequate to cognitive abilities. Skura and Lisicki (2015, p. 16) indicate that introducing mathematics in the cognitive development of a preschool child should take place through deduction (from the general to the specific), through encouraging experimentation with different strategies and ways of thinking. It should be emphasised that in this case, the active presence of the parent and the teacher, with the child on the path of development, is crucial because their approach to the child, their openness and readiness to arouse interest in mathematics, may determine his interest in mathematics, at a further stage of education. While dealing with mathematical education at the pre-elementary level, it is good to remember that a student, successfully ending elementary education, manifests mathematical culture if he/she is:

– mathematically proficient (S),
– has a formed language of mathematics at the level of bilateral communication (J),
– is characterized by developed spatial imagination, shows elements of mathematical creativity and elegance of reasoning.

The described model of mathematical culture considers components of mathematical competencies, useful not only in mathematical education or everyday life but first of all in the process of continuous cognitive development, which lasts throughout life and after completing formal education.

It should be stressed at the same time that a critical aspect and, at the same time, an advantage of developing a mathematical culture in children is creating the habit of constant learning and interpreting the world, discovering the unknown. The child's eagerness to explore previously unknown areas of science, the desire to understand new concepts is understood as curiosity about the world and the ability to look at objects and phenomena through the prism of mathematical knowledge (Krutiecki, 1971 p. 257–276). Giving mathematical meanings to objects and phenomena is possible when we refer to our knowledge, experiences. We communicate, express questions, doubts, answer and discuss.

Spatial orientation and mathematical culture of the preschool child

A correctly developed spatial orientation of a child is a foundation for the development of imagination in later years, both pictorial and verbal and constructional. Research shows that 65.4% of the 127 six-year-olds studied have a low operational reasoning level in determining the constancy of length

(Gruszczyk-Kolczyńska, 2008, p. 60) when observing changes in shape, position or form. “The consequence of the fact that school education does not attach importance to supporting children in these areas of orientation is that many pupils have excessive difficulties in the following years of education, not only in mathematics” (Gruszczyk-Kolczyńska, Makiewicz, Prończuk, 2021 p. 130).

A young child gradually gets to know the space, learns to move in it, and registers and communicates the relations between its elements. The research conducted by Gruszczyk-Kolczyńska shows that this process proceeds in stages: from child egocentrism, through gradual decentralisation, to marking out directions from a chosen object and determining the position of other objects to it, as well as the ability to orientate on a piece of paper and to switch between three-dimensional space and a flat (two-dimensional) piece of paper (Gruszczyk-Kolczyńska et al., 2021, p. 128–130).

Understanding the constancy of distance and the child's use of numbers as measures of distance and position is also related to the correct development of spatial orientation in the child. Children’s problems with understanding the meaning of length measurement are caused by underdeveloped operational reasoning at the specific level (Gruszczyk-Kolczyńska et. al., 2021 p. 139). For example, a child seeing the same strip of paper straightened and folded claims that the other one is shorter (photo 2).

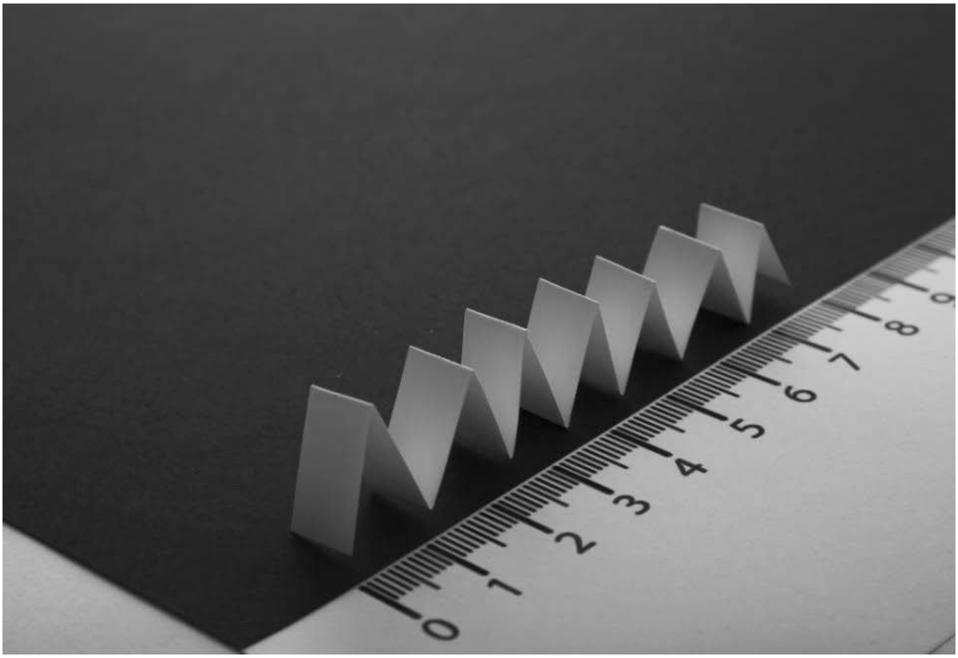

Photo 2. Length suggestion – Anna Panek-Kusz

Understanding of the constancy of length regardless of the form (curl, fold, etc.) does not develop until around the age of eight (Gruszczyk-Kolczyńska et al., 2021, p. 139). Sticks can also be used to diagnose the child's level of mental operations. The child has to compare the length of a segment made of 7 sticks and the length of a broken line (zigzag) made of 7 sticks. Gruszczyk-Kolczyńska emphasises that understanding the constancy of length is a prerequisite for orientation to common features of tools for measuring length (Gruszczyk-Kolczyńska et al., 2021, p. 140–141).

According to Piaget, until the age of two, the young child learns about the world mainly through movement and the senses. The child before the age of two is characterized by cognitive egocentrism. He perceives the environment from his point of view, unable to change his point of reference. Between eight and twelve months, he/she begins to understand the constancy of objects – objects still exist even if they cannot be seen. When an adult covers a teddy bear with a blanket, the child knows very well that the teddy bear has gone into hiding and that all he has to do is lift the blanket for it to reappear. It is a tremendous developmental achievement.

The pre-school child is in the period of development of pre-operational operations. In the months that follow, he discovers new possibilities for objects and experiments with different objects. A child discovers what happens when a toy is thrown far, how high a piece of furniture can be climbed, whether a block can be drowned and how loud a door slams. It is difficult for him to understand that the two strips of paper shown in the photograph below are the same length. Even if we checked the sameness of both strips of paper before his eyes and then folded one of them into an accordion, it is only in the early years of childhood that the child reaches a concrete level of operational thinking. In this context, it is essential to emphasise the parent, carer and teacher's vital role in helping the child activate and strengthen spatial orientation.

Gruszczyk-Kolczyńska points out that in this case, special attention should be paid to the development of the child's awareness of his/her body while:

– determining directions in the axis of the child's body,

– supporting the child in determining the position of objects concerning his/her body, with particular attention to the left and right sides.

There is no doubt about the usefulness of properly developed spatial orientation skills in the further education of a child, adolescent or adult. Regardless of age and level of education, a developed geometrical intuition or practical ability to read and interpret maps and technical drawings is highly advantageous. The author of "Child mathematics", referring to her long-term research, states the frequency of occurrence of mathematical talents in children in particular years of life: in 5-year-olds, every fifth child shows aptitude, in 6-year-olds it is every fourth child, and among first-graders aptitude is manifested only in every eighth pupil. Research has shown that at the turn of kindergarten and primary school, there is a crisis in the development of mathematical talents (Gruszczyk--Kolczyńska, 2012, p. 53). Considering the above factors and comparing them with the conditions of the Polish educational system, concern for the shaping of mathematical culture is justified from an early age.

Mathematical competencies in the context of the new core curriculum for pre-school education

The new core of the educational programme included in the documents of the 2017 reform of the Polish education system defines basic skills and concepts necessary for mathematical education at higher levels. The mathematical content of pre-school education specifies that the child, as a result of pre-school education:

1 Expresses creative expression during construction activities and play, arranges space by assigning meaning to objects placed in it, determines their location, number, shape, size, weight, compares objects in their environment according to selected characteristics.

2 Classifies objects by size, shape, colour, purpose, arranges objects in groups, rows, rhythms, recreates arrangements of objects and creates his own, giving them meaning, distinguishes basic geometric figures (circle, square, traingle, rectangle).

3 Experiments, estimates, predicts, measure the length of objects using, e.g. hand, foot, shoe.

4 Determines directions and establishes the position of objects concerning him/herself and to other objects, distinguishes left and right side.

5 Counts elements of a set during play, housework, exercises and other activities, use counters, recognize numbers from 0 to 10, experiments with forming consecutive numbers, perform addition and subtraction in a practical situation, counts objects, distinguishes between incorrect and correct counting.

6 Uses concepts of time sequence in play and other activities, e.g. yesterday, today, tomorrow, morning, evening, including names of seasons, days of the week and months.

7 Recognizes low-denomination coin and note models, arranges them, understands what money is used for in the household (Podstawa programowa…, 2017, p. 9).

It is worth noting that in the core mentioned above curriculum (points 3, 4), the legislator acknowledged the importance and value of spatial orientation issues in educating preschool children. Does the presented range of skills meet the legislator's intentions? An answer to this question may be provided by recalling the most important objectives of mathematics education – "developing the ability to learn about reality, making hypotheses, logical reasoning, effective use of various mathematical strategies and procedures, solving various problems" (Skura, Lisicki, 2015, p. 16).

Conclusion

Current Polish mathematics education has been characterised, in recent years, by high volatility. The unbelievable organisational effort connected with the necessity of implementing remote education has undoubtedly slowed down efforts to improve the quality of education. Given the above, it can be concluded that concern for the culture of the educational process, especially in mathematics, should be a point of interest for educational authorities.

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