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THE CHANGES OF AIR GAP IN INDUCTIVE ENGINES AS VIBRATION INDICATOR AIDED BY MATHEMATICAL MODEL AND ARTIFICIAL NEURAL NETWORK

Boguslaw Twarog

University of Rzeszow, Institute of Technology
35-959 Rzeszow, 16A Rejtana Str., Poland

Robert Pekala

University of Rzeszow, Institute of Technology
35-959 Rzeszow, 16A Rejtana Str., Poland

Jacek Bartman

University of Rzeszow, Institute of Technology
35-959 Rzeszow, 16A Rejtana Str., Poland

Zbigniew Gomolka

University of Rzeszow, Institute of Technology
35-959 Rzeszow, 16A Rejtana Str., Poland

Abstract. The method of analyzing vibration of electric engines or electromagnetic generators proposed in the work is based on the analyzing of course current of load. In considerations were used the method based on specialized mathematics model and advanced calculation technique. It allow to create of patterns for artificial neural networks. These patterns represented different states of machine for the diagnostic and they are enable to define precisely the changes caused by failure. Received experiments showed that the designed architecture of the net enables to achieve good properties of generalization correct answer for entrance date which weren't a part of training process.

1. **Introduction.** The suggested method for diagnosing the vibrations of inductive engines or generators is based on the analysis of the current load flow of those machines. In the dissertation a method based on specialistic mathematical model and advanced calculating technology leading to the creation of standards being the basis for diagnostic evaluation of a machine by artificial neural networks has been used. Those standards enable precise determination of the changes of the object in the confrontation with the measurement signals described by a given model. Such an approach is easy to algorithmize; it enables a constant generation of standards and a separate analysis of the particular types of the damages of the machine. Moreover, in connection with the fact that frequently the reasons for the damages overlap each other, the application of the artificial neural network as the right analytical tool seems to be justified. The conducted experiments showed that the designed network architecture enables to obtain good generalization properties, it

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generates correct answers for the output data which were not included in the content of the learning sequence. Taking into account the efficiency of working of the neural network as a quotient of correct answers to all possible answers, the efficiency for the assumed precision equaled 0.94, which can be considered a satisfactory result.

2. **The concept of the analytical system in real conditions.** In real diagnostic systems the measurement system has to show the current condition of the object. Usually in techniques we deal with two categories of the detection and diagnosis method: the estimation methods, the methods of model recognition. The estimation methods require the knowledge of the mathematical model which reflects the real process. The model cannot be too complicated so that solving of its equations does not last long because of the realization of the diagnostics in the real time. When a symptom of a faulty operation of the machine appears the measurement system should report it immediately and next the right diagnosis should determine the classification of the error and its location and if it is possible to isolate its influences. In the second category of the recognition methods there is not a mathematical model of the object or the process of diagnosing and classifying errors. It is a mapping of the area of the process from the measurements to the decision area.

The traditional recognition and classification is divided into three levels: measurement, analysis, classification. The artificial neural networks may realize the classification tasks in a very efficient way. It is connected with the choice of the architecture of the network which depends on the problem it is supposed to realize. In this case the selected features of the object decide about the number of neurons in the input, output and hidden layer. The structure of the network also depends on the complexity of the problem being solved by the network. The advantage of the neural network over the traditional methods of diagnosing results from the fact that it can generalize and if it is taught on the representative data it can function correctly even for cases which have not been discussed before.

2.1. **The architecture of the monitoring system.** The analysis of the object, in our case the electric engine, carried out with the help of this system may be used as a non- destructive method for monitoring the condition of the work of the object with the use of the artificial neural networks (fig. 1). It allows to discover the first symptoms of damages which has a significant meaning for providing a long lasting fault-free work. The suggested system consists of three main parts: the measurement system, the artificial intelligence system, the presentation system.

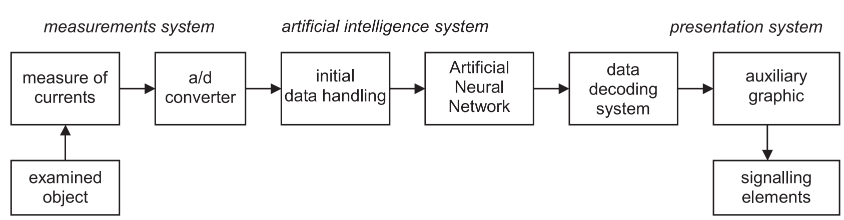

FIGURE 1. The modular architecture of the system of the monitoring of the object.

The realization of this measurement system (work in real time) has been carried out in the laboratory equipped with IBM PIV 2.0 computer together with the control - measurement module LC-012-1612, which enables carrying out measurements of the electric quantity or represented by physical non- quantity by means of the analogue - digital transducer. The advantage of the module is the possibility of retuning the range of input and output voltage, of which the correct setting may prevent from redirecting and at the same time it will enable the increase of the measurement precision.

3. **The dynamic mathematical model of the inductive engine used in the module of the artificial intelligence as a teacher of artificial neural networks.** The presented mathematical model was built with the use of the concept of special vectors of currents, voltage and magnetic streams because only they operate the first harmonic of the magnetic field in the air crack of the machine [1]. Moreover, two simplifications have been adopted during the consideration, that is omitting the losses connected with the hysteresis loop and in the laminated areas and skin currents, the division of the magnetic field of the machine into two - the main and diffused. In spite of the simplified consideration of the inductive engine as an electromechanical system, the mathematical model is still precise and complicated. It is caused, among others, by the change of the layout of the mathematical parameters in the system on account of the spinning of the rotor. Mutual inductive relations depend on the angle of the rotor spinning, that means the state equations become parametrical. However, the general theory of the differential equation claims that any parametrical equation with variable coefficients may be led to equations with constant coefficients. However, in this case it is necessary to find the transition matrix from one system into another. In the theory of rotation machines the transition matrix of the coordinates is found on the basis of uncomplicated geometrical transformations [1].

3.1. **The dynamic mathematical model of the inductive engine.** In the presented model the rotor is considered as an absolutely hard field with mass m and the inertion moment suitably to the center of the mass J. The misbalance which is created in the construction of the rotor is a result of the shift of the center of the mass c_m in comparison with its rotation axis c_R, which corresponds to the value ε in the adopted notations (fig.2). In fixed coordinates (x, y), the coordinates of the

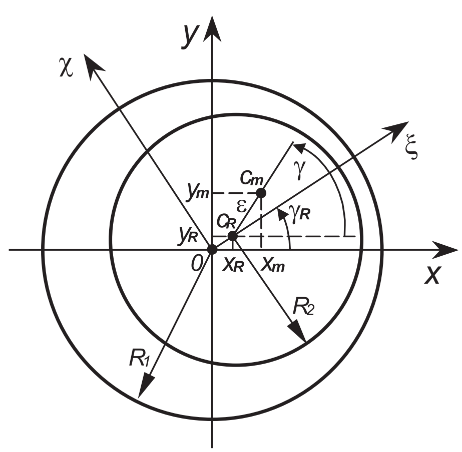

Figure 2. The graphical model of the rotor–stator system of the inductive engine with crosswise engine movements.

center of the rotor mass and its rotation axis is marked accordingly x_m, y_m and x_R=x_m-$\varepsilon\cos\gamma$, y_R=y_m-$\varepsilon\cos\gamma$. The angle of the rotation of the rotor γ_R , and γ the angle of the rotation of the coordinates system (ξ, χ) with the mutual center with the system (x, y), with the axis ξ coming through the geometrical center of the rotor. The changes of the air gap δ are described by the equation (1) obtained after suitable geometrical transformations:

$$\delta(\eta) = \sqrt{x_R^2 + y_R^2 + R_1^2 - 2R_1\sqrt{x_R^2 + y_R^2}\cos\eta} - R_2 \tag{1}$$

The result of the mathematical- technical considerations is the system of eleven differential equations (2) describing the dynamic states of the work of the inductive engine [2]. During the research on this model it turned out that the set of equations describing the model is a stiff system and the only way for the correct solving this type of equations is the application of secret methods of integrality. The stiffness

$$\begin{cases} \dot{\upsilon}_{xm} = F_x/m \\ \dot{\upsilon}_{ym} = F_y/m \\ \dot{\omega} = (M_E - M_M + F_x\varepsilon\sin\gamma - (F_y + mg)\varepsilon\cos\gamma)/J \\ \dot{\gamma}_R = \omega_R \\ \dot{x}_m = \upsilon_{xm} \\ \dot{y}_m = \upsilon_{ym} \\ \dot{\gamma} = \omega \\ \dot{\Psi}_{S\xi} = u_\xi + p_0\omega_R\Psi_{S\chi} - r_S i_{S\xi} \\ \dot{\Psi}_{S\chi} = u_\chi - p_0\omega_R\Psi_{S\xi} - r_S i_{S\chi} \\ \dot{\Psi}_{R\xi} = p_0\left(\omega_R - \omega\right)\Psi_{R\chi} - r_R i_{R\xi} \\ \dot{\Psi}_{R\chi} = p_0\left(\omega - \omega_R\right)\Psi_{R\xi} - r_R i_{R\chi} \end{cases} \tag{2}$$

in such electronic systems is connected, among others, with the occurrence of mechanical oscillations of high frequencies. To make the iteration process for stiff systems of equations correct taking the conditions of similarity into account, that means that to make sure the components which are changing the fastest do not limit the speed of the similarity and therefore the allowed length of the step, the Gear's method of the 6th class with the individual calculation of the Jacobian matrix has been applied [3]. Figures 3 and 4 show the simulation runs reflecting the character of the work of the inductive engine.

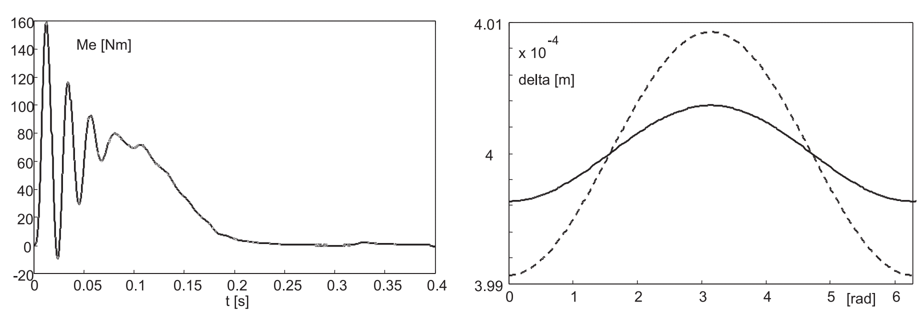

Figure 3. The time run of the changes of the electromagnetic moment and the size of the air gap (delta) rotor–stator of the inductive engine in different time periods (- t = 0.001s, - - - t = 0.15s) in the function of the rotation angle [0, 2π].

The special attention should be given to graphs in figure 4 which show sample time runs of the current of the stator of phase A for different eccentricity values of

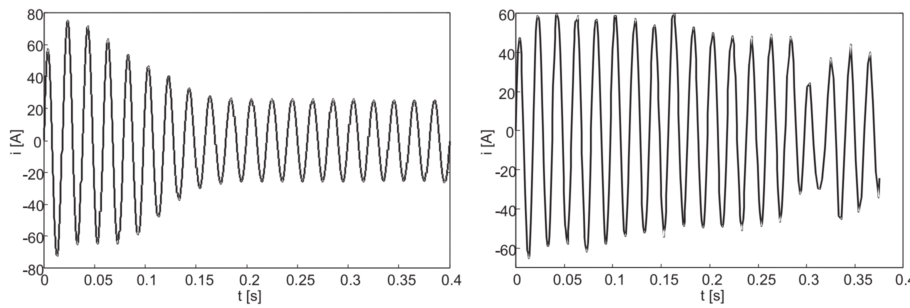

FIGURE 4. The time run of the current changes of the stator of phase A of the eccentricity of the rotor of the inductive engine ε=5.0e-05 i ε=2.0e-04.

the air gap of the rotor–stator system. The eccentricity of the position of the rotor of the inductive machine in relation to the stator desymmetrises the air gap δ (delta) (formula 1) and as a result of the characteristic changes in mutual magnetic couplings between the coils of the machine it influences the character of the current flows of the machine (fig. 4), in particular the currents of the stator in the set working condition [2].

3.2. **The system with the sigmoidal neural network.** Because of the lack of the occurrence of non- linear relations between the changes of the analyzed quantity and their causes the non-linear artificial neural networks have been used in the researches. Such tasks require solving the problem of the right transformation of the runs describing the object as well as carrying out experiments in the range of choosing the right architecture and data for teaching to obtain the ability of the network to generalize. The direct pass of the current flow on the receptors of the neural network is connected with the necessity of the digitizing of this flow, whereby the lowest the digitizing step the more precisely the flow will be represented. However, this kind of solution is not practical, because it leads to the excessive extension of the architecture of the neural network and it is connected with many inconveniences while realizing this teaching process. There is a necessity of the initial converting of the current flow stator, that is a modulated flow. In order to do this there has been the current distribution into the Fourier series applied. The steady flows of the engine current obtained from the mathematical model are the antisymmetric function, that is why there is not any component constant or even components in the Fourier's distribution. Thus, there are the amplitudes and phase angles of these components considered in the research, and their number is determined by the size of the single input vector, and the same time the number of receptors of the artificial neural network. The frequency value and the component amplitude of the particular spectrum are the result of the solution of the mathematical model equations for the steady work state. The quantitative evaluation of the spectrum will be done by using the neural networks. This kind of approach does not need the division of the spectrum of the stator current into particular component sets, characteristic of the particular eccentricity kind. At the neural input there is a vector given, including the amplitudes and particular phases of the components related to the value of the grooved component. Doing the diagnostic analysis of the induction motors with the eccentrically placed rotor on the basis of the solutions of the mathematical model equations was able to conduct after the meeting the following conditions:

- Possible, detailed description of the parameters of the model equation.
- Effective solution of the big number of the model equations.
- Precise evaluation of the obtained results.
- Full automation of calculations.

Figure 5 presents the value of the Ah amplitudes and the phase angles Fh (in radians) first 8 odd components of the current winding, received by different eccentricity values of the induction motor rotor. Taking the error value as the criterion of the components number selection, it was acknowledged that the first 4 odd components approximate the winding current with the sufficient accuracy, the approximation errors were of the value 10e-5. In connection with it, the input signal for the network is represented in the form of the 8 element vector made up of 4 amplitudes and 4 phase angle component solutions.

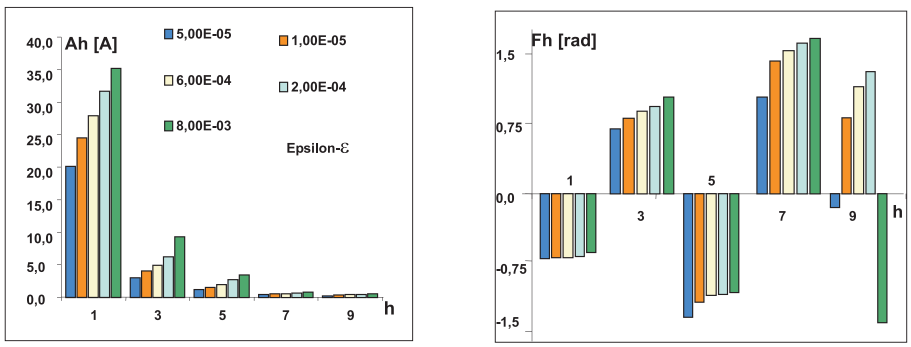

FIGURE 5. The Ah aplitude values and phase angles Fh of the current winding components for different eccentricity values (epsilon–ε) of the air–gap.

As we can notice, different winding eccentricity values generate different spectrums (amplitude values and phase angles) of the winding current components, that is why these changes can be used in research with the application of the artificial neural networks [4], [5]. In this case there has been the non-linear neural network applied for the purpose of the experiment made up of one hidden aspect. The neurons have been checked in this aspect with the sigmoidal and tangensoidal transfer function. In the inductive aspect there has been one neuron used because the task of the network was to identify only one eccentric value quantity of the inductive motor rotor. This neuron realized the linear function of the activation.

The number of the network receptors have been accepted according to the elements number in the inductive vector. For teaching the networks, the gradient method of Levenberg Marquardt has been applied in connection with the backpropagation algorithm, implemented in the programming language of Matlab for Windows.

In order to achieve the bigger accuracy of the gained results, the voting method has been applied, thanks to which the final result is achieved by most of the same answers generated by different neural networks [5]. The particular neural networks, apart from being taught with the same input vectors, achieve different levels of generalization. After checking the function of the system made up of five voters, it appeared that there has been the biggest accuracy and reliability of the results achieved (fig. 6).

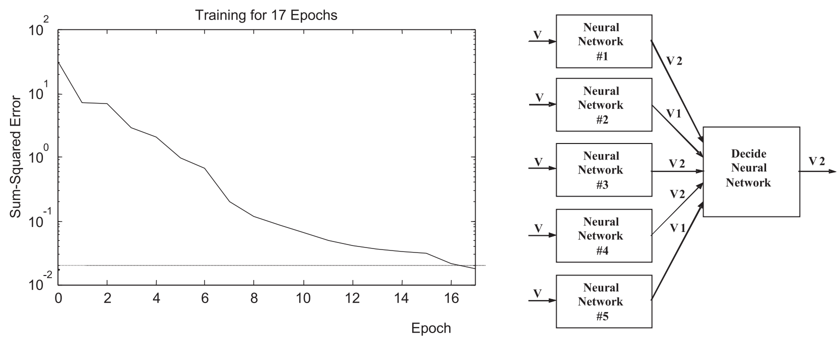

FIGURE 6. The course of teaching error in the function of the stages number necessary to achieve the established error level and the decision circuit combined of the five separate ANN.

For the final evaluation of the activity quality of the network there has been the independent, evaluation sequence used, made up of 80 input vectors. For both of these checked functions of the hidden neurons activation, the similar tests results have been achieved. Figure 7 in the table shows the tests results of the identification of the eccentricity value with the use of the network combined of 7 neurons in the hidden surface with the sigmoidal activation function. Columns Eps, EpsR include

No.	*Eps*	*EpsR*	No.	*Eps*	*EpsR*
	[m]	[m]		[m]	[m]
1.	1.10e-05	1.04e-05	11.	6.20e-05	6.16e-05
2.	1.30e-05	1.31e-05	12.	6.60e-05	6.67e-05
3.	2.20e-05	2.22e-05	13.	7.00e-05	7.08e-05
4.	2.50e-05	2.48e-05	14.	1.40e-04	1.46e-04
5.	3.30e-05	3.29e-05	15.	2.00e-04	2.07e-04
6.	3.60e-05	3.58e-05	16.	2.20e-04	2.28e-04
7.	4.20e-05	4.16e-05	17.	2.50e-04	2.58e-04
8.	4.70e-05	4.72e-05	18.	3.20e-04	3.27e-04
9.	5.10e-05	5.07e-05	19.	3.50e-04	3.58e-04
10.	5.50e-05	5.55e-05	20.	4.00e-04	4.09e-04

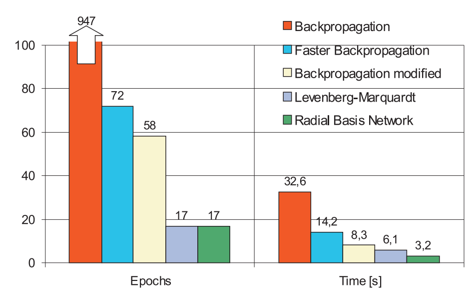

FIGURE 7. Results of the eccentricity value identification tests of the inductive motor rotor and the comparison of different types of artificial neural networks used in work, taking into consideration their speed of achieving the set teaching error on the level of 0.02.

appropriately the eccentricity value of the rotor given in the model and values corresponding the signals generated by the input neurons. Using the input neuron with the linear activation function causes that the received answers of the network express directly the eccentricity values of the induction motor rotor. All the collected results, put in the above table, allow to state that the used method is a good solution for the increasing the accuracy and above all the reliability of the received answers. Using this solution is profitable though it needs the sacrifice of the longer time for teaching and testing all networks.

In case of the research with the artificial neural network with the radial transfer function it allowed to identify the changes of the eccentricity of the induction motor rotor without the current flow distribution of the stator into the Fourier's series.

Its characteristics of functions allow for the classification of the particular parameter on the basis of the changes character of the flows including the essential information. Here such flows were the stator's currents i_A and i_B and they were the input signals to the radial neural network.

4. **Summary.** The main virtues of using the artificial neural networks in the diagnostic testing of the technical system result from the attractive network properties, such as the ability of the parallel information processing, effective approximation of free nonlinearity, teaching and tutoring the networks on the basis of the object signals. Moreover, using the specialized mathematical model and advanced computational technology led to the proper network training and allowed for the precise evaluation of the device state like the induction motor. Below there are the detailed conclusions that are the result of the task realization:

- The special approach has been proposed for the analysis of the dynamic stresses in the induction motors considering the mechanical and electromagnetic vibratory effects.
- The analytical relations have been received for the defining of the size of the air-gap in the induction motor that allow for the mutual interaction of the mechanical and electromagnetic subsystems in the motor,
- The mathematical models have been compared in terms of the usefulness in computer simulations,
- The effectiveness of the artificial neural network application has been shown for the analysis of the non-linear object state using among others: their virtues for example the abilities to generalize, low sensitivity on possible damages (breaking certain number of the joints between the neurons), the ability of the parallel processing and the work in the real mode.

REFERENCES

[1] V. Tchaban, "Mathematics modelling of electromechanical processes," LVIV, 1997.

[2] A. Tchaban and B. Twarog, *Simulation of lateral vibrations of induction motors*, Proceedings of International Conference on Modeling and Simulation, LVIV, (2001), 268–269.

[3] R. Slonevsky, A. Tchaban, O. Nechay, V. Tchaban and B. Twarog, *The Difficultes of Solution Very Stiff Differential Equations*, Proceeding of MS'2002. International Conference on Modelling and Simulation in Technical and Social Sciences, Girona, (2002), 909–916.

[4] S. Ossowski, "Sieci neuronowe do przetwarzania informacji", (in Polish), Wydawnictwo Politechniki Warszawskiej, Warsaw, 2000.

[5] J. Korbicz, J. Koscielny and Z. Kowalczuk, "Diagnostyka process. Modele. Metody sztucznej inteligencji. Zastosowania," (in Polish), WNT, Warsaw, 2002.

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E-mail address: btwarog@univ.rzeszow.pl